Robust Production Insuring Procurement and their Pitfalls*

Laurent Lamy† and Clément Leblanc‡

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Abstract

In a procurement setting involving both moral hazard and ex post risk where the contracting rule depends on realized production, we formalize a concept of robust insurance provision which reduces risk premiums with a prior-free approach. This leads us to analyze procurement where the auction-determined contract depends not only on the contractor's bid but also on a declaration on his expected production. For any given menu of linear contracts, we characterize the corresponding production-insuring menus and establish a general incentive to overstate expected production. We then analyse the pitfalls associated with false declarations in the lowest-price auction while putting aside moral hazard. We illustrate our analysis through simulations calibrated on a few offshore wind power auctions in France.

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Linear contracts as a function of output are widespread in practice. If the risk-neutral buyer values the contractor's output linearly, and if the contractor is risk neutral, then welfare-maximizing contracts take the form of linear contracts: Aligning the contractor's payoff with the buyer's private valuation of the output – also referred to as providing marginal rewards – is the only way to solve the moral hazard problem in a robust manner (Hatfield, Kojima and Kominers, 2018). When the buyer faces multiple firms among which she wants to select the most efficient contractor, the adverse selection problem can be solved with a second-price auction in which bidders compete for a fixed cash transfer in addition to a subsidy equal to the buyer's private valuation of

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†CIRED (Ecole des Ponts, ParisTech). Email: laurent.lamy78@gmail.com.
‡Universidad Carlos III de Madrid. Email: cl.clement.leblanc@gmail.com.

the output: this auction mechanism corresponds to the celebrated Vickrey-Clarke-Groves (VCG) mechanism, which not only implements the ex post efficient allocation in dominant strategies but also induces ex ante efficient private investments (Rogerson, 1992). The VCG mechanism is also known to maximize the principal's payoff in private value environments with endogenous entry (Jehiel and Lamy, 2018). Alternative theoretical approaches support the use of linear contracts by profit-maximizing principals while maintaining that the contractor is risk neutral and that the principal values output linearly: The optimal contract from the buyer's perspective is linear if she does not know the set of hidden actions available to the contractor and adopts a maxmin optimality criterion (Carroll, 2015). Dynamic moral hazard settings – where agents can adjust their effort to past performance – also provide support for linear contracting (Holmstrom and Milgrom (1987), Chassang (2015)).

If the contract involves ex post risk, then a risk-averse contractor would demand higher subsidies to obtain the same level of expected utility as a risk-neutral firm. In a competitive procurement setting, this translates into higher equilibrium prices and thus higher expected costs for the buyer, as analyzed by Eső and White (2004) in an auction setting.[1] Transferring an exogenous risk from the contractor to the risk-neutral buyer allows to reduce risk premiums and thus procurement costs (Engel, Fischer and Galetovic, 2013). As an example of the potential benefits of risk-sharing contracts, Engel, Fischer and Galetovic's (2001) seminal contribution about competitive concession awarding mechanisms pleads for least-present-value-of-revenue auctions in which the duration of the concession is adjusted to demand realizations: according to their calibration exercise for a highway franchising project in Chile, such risk-sharing contracts could reduce expected public spending by more than 20% compared to the widely used fixed-term contracts in which contractors bid on tolls for a fixed duration. Nevertheless, such contracts reduce the contractor's incentives to make costly efforts to increase demand as argued by Tirole (1997).

We consider a procurement setting with moral hazard and ex post risk: the contractor takes an unobserved costly action, which consists in choosing a project that stochastically determines a production in $\mathbb{R}_+$. Starting from a procurement where the buyer's payment to the contractor is a linear function of the realized production (that depends on the contractor's bid), we question whether it is relevant for the buyer to deviate from such menus of contracts to reduce the contractor's risk exposure. We address insurance provision while remaining agnostic about both the portfolio of projects available to each bidding firm and their risk preferences and cost functions. To our knowledge, our contribution is the first to address risk sharing in line with the so-called robust mechanism design and contracting literature (see Bergemann and Morris (2012) and Carroll (2019) for introductions to this flourishing topic).[2]

[1] See Vasserman and Watt (2021) for a survey of the theoretical and empirical work on auctions with risk aversion.

[2] In contrast, McAfee and McMillan (1986) address the trade-off between risk sharing, efficiency, and rent

The starting point of our analysis is the formalization of a general notion of robust insurance provision that compares contracts with a prior-free approach. We obtain then straightaway from the definition that it is impossible to design a contract that provides more insurance than a given contract for any possible distribution: having distributions with different expected production precludes the possibility of designing contracts that provide more insurance than a given one if the contracting rule does not depend on a declaration on the contractor's project. This impossibility does not solely justify the introduction of a declaration in addition to the price bids in the procurement rules, but also leads us to consider procurement where bidders declare their expected production.

Our work makes then two main theoretical contributions. First, we characterize the set of contracts that provide more insurance than a given linear contract (i.e., a contract that pays the contractor $\lambda \cdot q + \mu$ as a function of realized production $q \geq 0$) for two classes of possible distributions of ex post risk among distributions with a given expected production $\bar{q}$. If distributions are fully unrestricted, then production-insuring contracts take the form of the linear contracts $\lambda' \cdot q + (\lambda - \lambda') \cdot \bar{q}$ where the slope λ' is smaller than λ. If the set of possible distributions is restricted to symmetric and single-peaked distributions, then production-insuring contracts are those that satisfy a symmetry property around the expected production and that over-remunerate (resp. under-remunerate) production for realizations that are smaller (resp. greater) than the expected production $\bar{q}$. Second, we establish that such production-insuring menus of contracts suffer from a manipulation concern if some firms – referred to as strategic firms – report the expected production that maximizes their payoff rather than their actual one: a firm whose risk aversion is limited would always benefit from overstating its expected production. Furthermore, such misreporting is shown to be detrimental to the buyer if the firm becomes the contractor, insofar as the buyer will raise a lower expected payoff that the one she would have expected to raise with the same offer but stemming from a truthful contractor. This vulnerability does not hold solely for some specific distributions but for any distribution against which the production-insuring menu of contract is presumed to hedge.

To analyze the consequences from such manipulation, we consider a setting without moral hazard where the portfolio of projects available to all firms is a singleton, e.g. where the decision of each bidding firm in the procurement reduces to a bid and a declaration on his project. For the winner-determination rule, we consider the lowest-price auction where the firm having submitted

minimization in a Bayesian manner. See also Laffont and Tirole (1986) and McAfee and McMillan (1987) for the characterization of optimal procurement with an approach à la Myerson (1981) when firms are risk neutral: optimal procurement can be implemented as a menu of linear contracts where agents self-select as a function of their one-dimensional private signals. In contrast, optimal contracting under multidimensional private information requires nonlinear pricing under Bayesian approaches (see Rochet and Stole (2003) for a survey on multidimensional screening).

the lowest bid becomes the contractor. Adopting a production-insuring menu (e.g., regarding the set of symmetric and single-peaked distributions) involves then a trade-off between two effects: On the one hand, it lowers the zero-profit bid of each firm, i.e. the bid that makes the firm indifferent between winning and losing the procurement auction, and then the equilibrium price compared to the linear benchmark of reference. Lower zero-profit bids reflect both reduced risk premiums when firms truthfully report their expected production and also the contractor's benefits from overstating his expected production. On the other hand, if the contractor is a strategic firm, then the equilibrium price does not reflect properly the cost that the buyer will have to pay on average ex post. We analyze this trade-off under some restrictions regarding bidders' beliefs on their competitors. In particular, we show that the gains from lower prices are always delusive when firms are symmetric and risk neutral. However, in general, the impact of adopting a production-insuring menu is an empirical question. We illustrate our insights with some offshore wind power auctions in France where the procurement rules involve a production-insuring menu of contracts. According to our estimates, the potential losses resulting from false declarations are 15 times greater than the potential benefits from reduced risk premiums under truthful reporting.

The remainder of the paper is organized as follows. Section 1 sketches the main lines of our analysis in a simplified environment. Section 2 presents the contracting environment with moral hazard and ex post production risk and introduces the novel concept of production-insuring contracts that hedge the contractor against production risk for any distribution in a given set of production distributions (in comparison to a contract of reference). Section 3 presents the procurement game where bidders make a declaration regarding the characteristics of their project on top of their bid. We introduce the concept of production-insuring menus of contracts (in comparison to a given procurement design) where each contract in the menu is production-insuring with regards to the set of production distributions associated with the same declaration. We consider furthermore a setting where the buyer may suffer from a lack of screening ability insofar as some firms are free to misreport their project characteristics in their declaration. The core of the paper is Section 4 which is devoted to production-insuring menus based on expected production and in comparison to menus of linear contracts that do not depend on any declaration. We start with a discussion on the optimality status of such menus of linear contracts under risk neutrality, which justifies its use as a benchmark of reference. We then characterize production-insuring contracts and analyse bidders' incentives to misreport their expected production and the associated deception for the buyer. Section 5 analyzes the consequence on the buyer's expected cost of using menus that are manipulable in a setting without moral hazard and when bidders differ only in terms of their ability to make false declarations on their project. A calibration is developed in Section 6 to illustrate the potential gains and pitfalls of a specific production-insuring menu that has been used in France for some large power auctions. Extensions of the model are discussed in Section 7, related literature is discussed in Section 8, and concluding remarks are made in Section 9. The proofs of our main results are presented in the Appendix while additional materials are available in a Supplementary Appendix (henceforth the SA).

1 Illustration in a simplified environment

Consider a risk-neutral buyer willing to pay a risk-averse contractor to build a project whose output $q \in \mathbb{R}_+$ is subject to ex post risk. The distribution of q, denoted by the PDF f, depends on the contractor's technology, which is unobserved by the buyer. Suppose the buyer values the contractor's output linearly, at v per unit. If $T : \mathbb{R}_+ \mapsto \mathbb{R}$ is the contract that specifies the buyer's payment to the contractor as an increasing function of his output q, then the buyer's expected payoff is equal to $\mathbb{E}_f[v \cdot q - T(q)]$. Since the buyer is risk neutral, she is willing to take the risk for herself in order to make the contractor better off, and ultimately to bargain for a lower expected payment, or to attract lower bids in a competitive procurement setting.

Take as a benchmark the linear contract $T(q) = \lambda \cdot q$ with $\lambda = v$, which aligns the contractor's revenue with his contribution to social welfare. The buyer might be tempted to design an alternative contract T' that would be strictly preferred by any strictly risk-averse contractor (i.e., for any strictly concave utility function U):

$$\mathbb{E}_f[U(T'(q))] > \mathbb{E}_f[U(T(q))] \tag{1}$$

while preserving the same expected payment made by the buyer:

$$\mathbb{E}_f[T'(q)] = \mathbb{E}_f[T(q)] = \lambda \cdot \mathbb{E}_f[q]. \tag{2}$$

One way to do this would be to switch to a "flatter" linear contract $T'(q) = \lambda' \cdot q + \mu$ by lowering the per-unit payment, i.e. $\lambda' < \lambda$, while compensating with a cash transfer μ. Setting the cash compensation at $\mu = (\lambda - \lambda') \cdot \mathbb{E}_f[q]$ guarantees the same expected payment, i.e. that (2) is satisfied. The inequality $\lambda' < \lambda$, which means that the risk associated with the realization of q is reduced, further implies that (1) is satisfied. However, the design of such a "production insuring" contract depends on the expected production $\mathbb{E}_f[q]$, which here may be private information of the contractor.

Suppose for simplicity that the contractor is either of low type, with expected production $\bar{q}_L$, or of high type, with expected production $\bar{q}_H > \bar{q}_L$. Applying the above idea leads to offering a binary menu of "flatter" linear contracts, $T'_{\bar{q}_L}$ and $T'_{\bar{q}_H}$, having in mind that the contract $T'_{\bar{q}_k}$ is suitable for the contractor with expected production $\bar{q}_k$ (for $k = L, H$). For a given per-unit payment $\lambda' < \lambda$, the corresponding cash compensations $\mu_{\bar{q}_k}$ should be set so that $\mu_{\bar{q}_k} = (\lambda - \lambda') \cdot \bar{q}_k$

in order to satisfy (2). If the contractor's type is not verifiable – and if λ' is the same for both contracts[3] – then there is a clear incentive for a low-type contractor to choose the contract $T'_{\bar{q}_H}$ (i.e., to report a high type), which will increase his payment for any possible production realization q (as illustrated in the left panel of Figure 1).

In this paper, we show that no contract can satisfy both (1) and (2) for any possible distribution, and that the linear contracts $\lambda' \cdot q + (\lambda - \lambda') \cdot \bar{q}$ with $\lambda' < \lambda$ are the only contracts that satisfy (1) and (2) for all distributions whose expected value is $\bar{q}$. Furthermore, we establish that when faced with a menu of such contracts, a risk-neutral firm or a firm with low risk aversion will always increase its expected payoff by reporting a type higher than its true type. In other words, these firms have an incentive to overstate their expected production. However, reducing the set of distributions over which we impose that the conditions (1) and (2) are satisfied for a given expected value $\bar{q}$ allows for a larger class of production-insuring contracts. In particular, with the only prior that the distribution of q is symmetric and single-peaked, the buyer can design such contracts simply by ensuring that they: 1. Increase the payment to the contractor for below-average realizations of q (and conversely decrease the payment for above-average realizations of q), relative to the benchmark T, and, 2. Preserve the buyer's expected payment to the contractor through a symmetry condition around the expected production $\bar{q}$. See the right panel of Figure 1b for an illustration with the green (resp. red) color for the low (resp. high) type contract. In general, misreporting incentives are less straightforward with such potentially nonlinear contracts. When switching from $T'_{\bar{q}_L}$ to $T'_{\bar{q}_H}$, the gains for realizations of q in the interval $[\bar{q}_L, \bar{q}_H]$, where we have $T'_{\bar{q}_H}(q) \geq T'_{\bar{q}_L}(q)$, could be offset by the losses outside this interval. However, a key result of this paper is that any menu of contracts that provides insurance in a prior-free way with respect to the set of symmetric and single-peaked distributions induces the contractor to overstate his expected production, as long as his risk aversion is not too high. This further implies that the buyer suffers from a form of deception: on average, she pays more than she would have paid if the contractor had been truthful.

This incompatibility between robust insurance and incentives has specific consequences in a procurement setting. To illustrate, consider the case of two payoff-symmetric, low-type firms competing in the VCG mechanism, i.e., in the present setting, where the firm with the lowest bid becomes the contractor with a contract $T_b(q) := v \cdot q + b$, where b is the second-lowest bid. If firms are risk neutral, it is a weakly dominant strategy for them to bid their zero-profit bid $c - v \cdot \bar{q}_L$, i.e. their cost c to build the project minus their expected revenue. If instead firms are risk-averse, their equilibrium bid b_U^{eq} is characterized by the zero-profit condition $\mathbb{E}_f[U(v \cdot \bar{q}_L + b_U^{eq})] = U(c)$, where U is their (concave) utility function. The concavity of U implies that b_U^{eq} is greater than $c - v \cdot \bar{q}_L$

[3]If the contract is steeper for the high type than for the low type, $\lambda'_H > \lambda'_L$ a risk-averse contractor faces a trade-off between increasing his expected payment by choosing $T'_{\bar{q}_H}$ and decreasing his risk by choosing $T'_{\bar{q}_L}$.

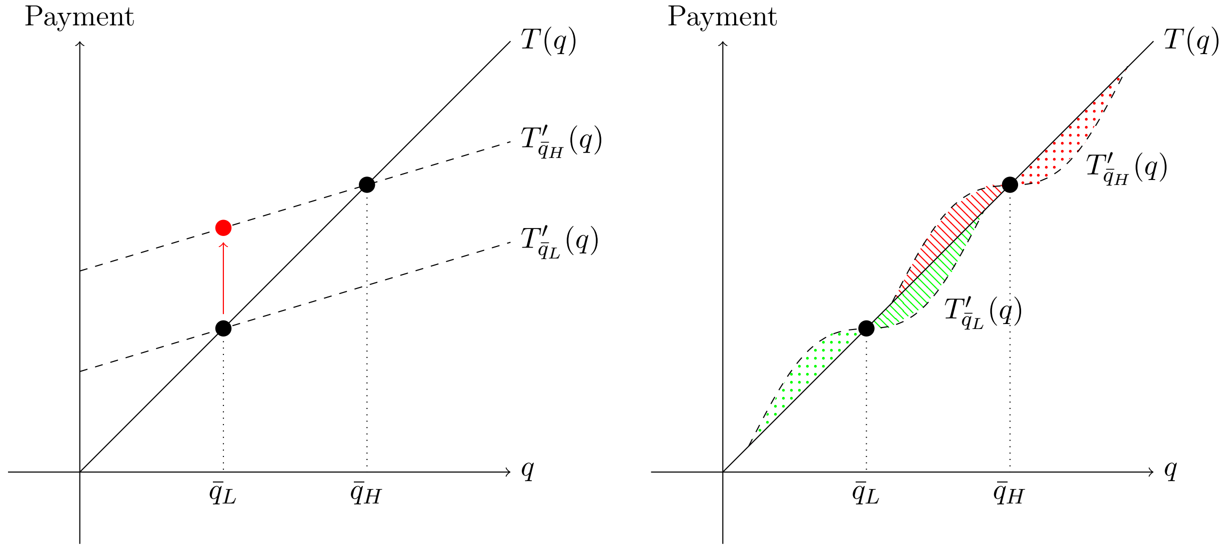


Figure 1: Contracts that transfer risks from the contractor to the buyer

which reflects the risk premium associated with risk aversion. To mitigate this risk premium, the buyer may decide to replace the menu of linear contracts T_b with a menu of production-insuring contracts $T'_{b,\bar{q}}(q)$ as discussed above: for any given bid b, firms would necessarily raise a higher expected payoff under this latter contract, allowing them to bid lower while maintaining a positive payoff. As a consequence of (2), the lower equilibrium bids result in a lower expected payment to the contractor if both firms truthfully report that they have a low type.

The picture is very different if the contractor has strategically reported a high type. To illustrate what may happen, suppose the buyer implements one of the menu of "flatter" linear contracts discussed above $T'_{b,\bar{q}}(q) = \lambda' \cdot q + (v - \lambda') \cdot \bar{q} + b$, with $\lambda' < v$, and that the firms are risk neutral. If the contract is $T'_{b,\bar{q}_H}$ and if the buyer believes that the contractor is of high type, then she expects to pay on average $v \cdot \bar{q}_H + b$ to the contractor for the expected production $\bar{q}_H$ and thus expects a payoff $-b$, but ends up with a payoff $-b - (v - \lambda') \cdot (\bar{q}_H - \bar{q}_L)$. Because of this form of deception, the lower equilibrium bids do not translate into a higher expected payoff for the buyer. The equilibrium bid depends actually on the strategic/truthful behavior of the firms. If both firms are strategic in reporting their type, then the equilibrium bid is $c - v \cdot \bar{q}_L - (v - \lambda') \cdot (\bar{q}_H - \bar{q}_L)$, which gives the buyer the same payoff as under the baseline menu of contracts T_b, i.e., $v \cdot \bar{q}_L - c$. However, if a single firm is strategic, the equilibrium bid is fixed by the truthful firm and is thus the same as under the baseline menu of contracts, i.e., $c - v \cdot \bar{q}_L$. The strategic firm then becomes the contractor and reaps a manipulation rent at the expense of the buyer, whose payoff falls to $v \cdot \bar{q}_L - c - (v - \lambda') \cdot (\bar{q}_H - \bar{q}_L)$. When risk aversion is introduced, the outcome is not as clear-cut and will be illustrated in our calibration exercise.

2 The contracting environment

Let us now turn to the more general model examined in this paper. A buyer organizes a procurement auction to select a contractor to develop an indivisible risky project. The performance of the project is characterized by the ex post publicly observable and contractible variable $q \in \mathbb{R}_+$, hereafter referred to as the production.[4] We consider a setting where output q results both from an unobservable investment made by the contractor and from an exogenous random shock.

Projects: A project, denoted by f, corresponds to a PDF on a compact subset of $\mathbb{R}_+$. Let F denote the corresponding (atomless) CDF, $\mathcal{F}$ the portfolio of possible projects from the buyer's perspective, and, for each firm i, $\mathcal{F}_i \subseteq \mathcal{F}$ the finite set of projects it can actually choose. Once the contractor i (secretly) chooses a project $f \in \mathcal{F}_i$, his production q is distributed according to the PDF f. We use the shortcut notation $\bar{q}_f \equiv \mathbb{E}_f[q] > 0$ for the expected production (or just $\bar{q}$ if there is no ambiguity about the distribution being discussed).

Contracts: A contract between the buyer and the contractor is a continuously increasing function $T : \mathbb{R}_+ \mapsto \mathbb{R}$, which maps a production q by the contractor to a payment $T(q)$ by the buyer. Since we assume that there are no additional costs associated with production once the contractor has chosen his project, the strict monotonicity of T guarantees that the contractor would always lose from rationing his production ex post.

The buyer's preferences: If the monetary transfer to the contractor is t and the quantity produced is q, the buyer's payoff is assumed to be $v \cdot q - t$, where $v > 0$ denotes the buyer's valuation per unit. The buyer is assumed to be risk neutral, so if the contract is T and the contractor chooses project f, then the buyer's expected payoff is $v \cdot \mathbb{E}_f[q] - \mathbb{E}_f[T(q)]$.

Firms' payoff characteristics: Consider $N \geq 2$ competing firms. Each firm $i = 1, \ldots, N$ is characterized by its (non-empty) private portfolio of projects $\mathcal{F}_i$, a cost function $C_i : \mathcal{F}_i \mapsto \mathbb{R}_+$, and a continuously differentiable increasing utility function $U_i : \mathbb{R} \mapsto \mathbb{R}$ with the normalization $U_i(0) = 0$. To capture risk aversion, we further assume that the utility functions U_i, $i = 1, \ldots, N$, are concave. We say that a firm with utility function U is risk neutral if U is linear, and risk averse (resp. strictly risk averse) if U is concave (resp. strictly concave).[5] Last, we say that firms are payoff-symmetric if both C_i and U_i do not depend on i. Under contract T, the expected payoff of contractor i implementing project f is $\mathbb{E}_f[U_i(T(q) - C_i(f))]$. Losing the auction and not implementing a project results in a payoff of $U_i(0) = 0$.

Social welfare: If firms are risk neutral, then the (utilitarian) social welfare does not depend on the monetary transfer between the buyer and the contractor: if the buyer contracts

[4]The variable q could also correspond to a measure of quality or, more generally, to any kind of one-dimensional verifiable measure characterizing the contractor's output.

[5]We say that a function $U : \mathbb{R} \mapsto \mathbb{R}$ is strictly concave if $U'(x) < 0$ for any $x \in \mathbb{R}$.

with firm i and the latter chooses project f, then social welfare is equal to $v \cdot \bar{q}_f - C_i(f)$. For each firm i, let $\mathcal{F}_i^* := \operatorname{Arg\,max}_{f \in \mathcal{F}_i} \{v \cdot \bar{q}_f - C_i(f)\}$ denote the set of socially optimal projects when firm i is the contractor, $f_i^* \in \mathcal{F}_i^*$ is a corresponding socially optimal project, and $SW_i^* := \max_{f \in \mathcal{F}_i} \{v \cdot \bar{q}_f - C_i(f)\}$ is the corresponding social welfare. A firm $i^* \in \operatorname{Arg\,max}_{i=1,\ldots,N} SW_i^*$ is called a (socially) optimal firm. We further denote $SW^* := \max_{i=1,\ldots,N} SW_i^*$ as the welfare associated with an optimal project among all firms, and $SW_{-i}^* := \max_{j \neq i} SW_j^*$ as the welfare associated with an optimal project among the competitors of firm i.[6] It is optimal to develop a project under the condition that $SW^* < SW^{NO}$, where SW^{NO} is the buyer's payoff if no project is developed. For simplicity, we always assume that $SW_i^* > SW^{NO}$ for each firm i.

Throughout our analysis, we mainly consider that the buyer's objective is to maximize her expected payoff, an objective that is actually congruent with social welfare under additional assumptions, as discussed in Section 4.1. In contrast to the optimal auction literature à la Myerson (1981), we do not specify the buyer's beliefs about firms' characteristics in a Bayesian manner. Rather, our approach belongs to the *robust mechanism design* paradigm: we wish to design contracts that perform well for any possible Bayesian specification of the buyer's beliefs about the distribution of firms' characteristics, and this given that the buyer knows that the contractor's project necessarily belongs to the portfolio of possible project $\mathcal{F}$.

We now introduce a partial order that compares contracts in terms of insurance provision.

Definition 1. *Take $\widehat{\mathcal{F}} \subseteq \mathcal{F}$. Consider two contracts $T(.)$ and $T'(.)$ such that $\mathbb{E}_f[T(q)] = \mathbb{E}_f[T'(q)]$ for any $f \in \widehat{\mathcal{F}}$. We say that $T'(.)$ provides more insurance than $T(.)$ on $\widehat{\mathcal{F}}$ if*

$$\mathbb{E}_f[U(T'(q))] \geq \mathbb{E}_f[U(T(q))] \tag{3}$$

for any $f \in \widehat{\mathcal{F}}$ and any concave utility function U, and if the inequality is strict for any strictly concave utility function U.

According to this definition, a contract that provides more insurance than another on a given subset of projects leaves the risk-neutral buyer indifferent, but is preferred (resp. strictly preferred) by a risk-averse (resp. strictly risk-averse) contractor for any project choice in the given set $\widehat{\mathcal{F}}$.

The existence of contracts that provide more insurance than a given one depends crucially on how large the set $\widehat{\mathcal{F}}$ is. If we take $\widehat{\mathcal{F}}$ to be the full set of atomless distributions whose support is a compact subset of $\mathbb{R}_+$, a set denoted next by $\mathcal{F}_{all}$, then no such contracts exists.

Proposition 1. *For any possible contract T, there is no contract that provides more insurance than T on $\mathcal{F}_{all}$.*

[6] We naturally have $SW^* \geq SW_{-i^*}^*$.

Proof: For any given $\widehat{q} > 0$, we can construct a sequence of distributions $\{f_n\}_{n\in\mathbb{N}}$ such that $f_n \in \mathcal{F}_{all}$ converges to the Dirac measure concentrated at $\widehat{q}$. The equality $\mathbb{E}_{f_n}[T(q)] = \mathbb{E}_{f_n}[T'(q)]$ for any $n \in \mathbb{N}$ implies that $T'(\widehat{q}) = T(\widehat{q})$. This equality for any $\widehat{q} \in \mathbb{R}_+$ then precludes any strict version of the inequality (3) when U is strictly concave. **Q.E.D.**

It is straightforward that the same reasoning applies to many subsets of $\mathcal{F}_{all}$, including the set of symmetric and single-peaked distributions $\mathcal{F}_{sym}$ (which will be discussed extensively in the remainder of the paper),[7] and also various parametric subsets of it, such as the uniform or the symmetric triangular distributions. It also extends to subsets of the above sets that include all projects whose expected production belongs to a non-degenerate interval $[q_{min}, q_{max}]$, i.e., with $q_{max} > q_{min}$. Formally, we obtain that $T'(q) = T(q)$ for any $q \in [q_{min}, q_{max}]$, so that inequality (3) stands as an equality for any distribution whose support is a subset of $[q_{min}, q_{max}]$, regardless of the utility function U.

Such impossibility results invite us to consider contracts that, in order to provide more insurance, would have to depend on some characteristics of the project: Insurance provision by a single contract design can typically be achieved only on a subset of the portfolio of available projects $\mathcal{F}$, one in which projects share some common characteristics. In particular, the latter result above suggests that projects with different expected production should be treated with different contracts. This leads us to introduce procurement games in which firms submit not only a monetary bid but also a declaration about some characteristics of their project. The next section remains very general about the nature of the declaration, while the remainder of the paper will consider procurement games where the declaration corresponds to the expected production of the project.

3 The Procurement Game with Partial Screening

Let us introduce Σ, a partition of $\mathcal{F}$ used to parameterize a menu of contracts offered to firms in a procurement auction. Let $\tilde{\sigma} : \mathcal{F} \mapsto \Sigma$ be the function that maps each project f to the subset in the partition Σ that contains f. The finest (resp. coarsest) partition corresponds to the case where $\tilde{\sigma}(f) = \{f\}$ (resp. $\tilde{\sigma}(f) = \mathcal{F}$) for each $f \in \mathcal{F}$. The partition Σ is related to the ability of the buyer to verify the characteristics of the project f to be implemented, and then to constrain (at least some) firms to report $\tilde{\sigma}(f)$ when choosing the project f. In the following, we distinguish between firms that are subject to such a constraint, called *truthful firms*, and those that are exempt from it, called *strategic firms*.

In the procurement game, each firm is invited to submit a *bid* $b \in \mathbb{R}$ and a *declaration* $\sigma \in \Sigma$ about its project. The applicable contract then depends on the contractor's bid and declaration,

[7] Formally, a PDF $f \in \mathcal{F}_{sym}$ satisfies: $f(\bar{q} + x) = f(\bar{q} - x)$ for any $x \in [0, \bar{q}]$, $f(q) = 0$ for $q > 2\bar{q}$ and f is non-decreasing on $[0, \bar{q}]$ and non-increasing on $[\bar{q}, 2\bar{q}]$.

to be picked from a menu of contracts $\mathcal{M} := \{T_{(b,\sigma)}\}_{(b,\sigma)\in\mathbb{R}\times\Sigma}$. We assume throughout our analysis that the function $b \mapsto T_{(b,\sigma)}(q)$ is continuously increasing for any $q > 0$ and any $\sigma \in \Sigma$, and that for any firm $i = 1, \ldots, N$ and any project $f \in \mathcal{F}_i$, $T_{(b,\sigma)}(q) - C_i(f)$ is strictly negative (resp. positive) if b is small (resp. high) enough. A firm is said to make the *decision* $d = (b, f, \sigma)$ when it decides to choose project $f \in \mathcal{F}_i$, to declare $\sigma \in \Sigma$ about its project, and to bid $b \in \mathbb{R}$. Let $\Pi_i(d) := \mathbb{E}_f[U_i(T_{(b,\sigma)}(q) - C_i(f))]$ denote the expected payoff of firm i conditional on winning with decision d. It follows from the restrictions we imposed on the menu of contracts $\mathcal{M}$ that for each pair $(f, \sigma) \in \mathcal{F}_i \times \Sigma$, the function $b \mapsto \Pi_i(b, f, \sigma)$ is increasing, and that $\Pi_i(b, f, \sigma)$ is strictly negative (resp. strictly positive) if b is small (resp. high) enough. We always consider *pay-as-bid* auctions, where the contract signed by the winning firm is the contract $T_{(b,\sigma)}$, where b is the winning firm's bid (or winning bid) and σ is its declaration. Since we restrict our attention to pay-as-bid auctions, for a given bid b, a firm's incentives with respect to its project choice f and its declaration σ do not depend on its beliefs about the characteristics of its competitors. Without further restrictions (e.g., risk neutrality), this separability property would not hold under alternative pricing rules (e.g., second-price auctions). For the winner-determination rule, we consider by default the *lowest-price auction*, where the buyer selects the firm with the lowest bid (possibly with a reserve above which the procurement remains unsuccessful) independently of firms' declarations about their project. Alternative winner-determination rules are discussed in Section 7.

We consider then two types of firms. The so-called *strategic* firms are assumed to be free to make any decision in $\mathbb{R} \times \Sigma \times \mathcal{F}_i$. On the contrary, the so-called *truthful* firms are constrained to make a declaration σ that matches their project choice f, i.e. their decision is constrained by $\sigma = \tilde{\sigma}(f)$. We denote $k \in \{T, S\}$ the type of a firm, with $k = S$ if the firm is strategic and $k = T$ if the firm is truthful. We assume that all firms, whether strategic or truthful, maximize their expected payoff. Consider a firm i that has submitted the bid $b \in \mathbb{R}$. If it is truthful, it will choose a project $f^* \in \operatorname{Arg\,max}_{f\in\mathcal{F}_i} \Pi_i(b, f, \tilde{\sigma}(f)) \equiv f_i^T(b)$ and submit the declaration $\tilde{\sigma}(f^*)$. If it is strategic, then it will choose a pair of project and declaration $(f^*, \sigma^*) \in \operatorname{Arg\,max}_{(f,\sigma)\in\mathcal{F}_i\times\Sigma} \Pi_i(b, f, \sigma) \equiv Q_i^*(b)$. Let $\Pi_i^T(b)$ (resp. $\Pi_i^S(b)$) denote the corresponding expected payoff of firm i conditional on winning with bid b if i is truthful (resp. strategic). Formally, we have then[8]

$$\Pi_i^T(b) := \max_{f\in\mathcal{F}_i} \Pi_i(b, f, \tilde{\sigma}(f)) \qquad \text{and} \qquad \Pi_i^S(b) := \max_{(f,\sigma)\in\mathcal{F}_i\times\Sigma} \Pi_i(b, f, \sigma).$$

Since the set of decisions available to truthful firms is a subset of that available to strategic firms, it

[8]Throughout our analysis, we implicitly assume that these maximization programs (as well as the welfare maximization programs presented earlier) have a genuine solution. This is straightforward if $\mathcal{F}_i$ is finite. In general, it would be necessary to impose more structure (e.g., compactness of the set $\mathcal{F}_i$ and continuity of the cost function and the menu of contracts).

is straightforward that $\Pi_i^S(b) \geq \Pi_i^T(b)$ for any bid b and any firm i. As a maximum of continuously increasing functions that cross zero, the function $b \mapsto \Pi_i^k(b)$ is continuously increasing for both $k = T$ and $k = S$, and is positive if b is large enough. Let us assume that $\Pi_i^k(b) < 0$ if b is small enough such that there exists a (unique) zero-profit bid denoted by $\widehat{b}_i^k$ and characterized by $\Pi_i^k(\widehat{b}_i^k) = 0$. We also use the notation $\widehat{b}_i^k(\mathcal{M})$ when we want to make explicit the dependence of the zero-profit bids on the menu of contracts. Since $\Pi_i^S(b) \geq \Pi_i^T(b)$ for any bid b, we have $\widehat{b}_i^S \leq \widehat{b}_i^T$ and the inequality is strict if the former inequality is always strict. In words, strategic bidders are prone to outbid truthful bidders, ceteris paribus.

The relevant specification for Σ is closely tied to the strategic/truthful dichotomy we introduce. The implicit idea motivating our analysis is that the buyer is able to verify that some firms (the truthful ones) make a reliable declaration about the project they intend to implement, but, on the contrary, is unable to screen some other firms (the strategic ones). The concept of a truthful firm is thus only relevant if the partition Σ is not too fine: we have in mind environments where it would be unrealistic to assume that the buyer is able to screen all the details of the project.

Timing of the game

1. The buyer commits to the procurement game (i.e., the menu of contracts $\mathcal{M}$ and the winner-determination rule).

2. Each firm $i = 1, \ldots, N$ learns its payoff characteristics $\mathcal{F}_i$, C_i and U_i and whether it is truthful or strategic.

3. Each firm i makes a decision d_i, i.e., submits a bid b_i, (secretly) chooses a project $f_i \in \mathcal{F}_i$, and makes a declaration $\sigma_i \in \Sigma$ to the buyer.[9]

4. The winning firm i^w is selected according to the winner-determination rule and becomes the contractor with the contract $T_{(b_{i^w}, \sigma_{i^w})}$. Under the lowest-price auction, we have $i^w \in \operatorname{Arg\,min}_{i=1,\ldots,N} b_i$).

5. The contractor pays the sunk cost $C_{i^w}(f_{i^w})$ to implement the project f_{i^w} and the production q is drawn according to the distribution f_{i^w}. The contractor receives the amount $T_{(b_{i^w}, \sigma_{i^w})}(q)$ from the buyer.

The equilibrium concept: Without further restrictions, pay-as-bid auctions are known to suffer from equilibrium multiplicity, especially under complete information.[10] We thus restrict our equilibrium analysis to undominated strategies. In particular, we ignore equilibria in which

[9]Since the choice of the project is secret, our analysis extends to environments where this choice is made at the beginning of stage 5. In such a case, the choice of a truthful contractor is bound by his declaration.

[10]In standard first-price auctions under complete information, any price between the highest and second-highest valuation can be sustained in equilibrium.

firms, because they expect to be outbid for sure, make decisions that would generate an expected negative payoff conditional on winning.[11] Thus, we assume that the equilibrium bid b_i submitted by a type-k firm i ($k = T, S$) satisfies $\Pi^k(b_i) \geq 0$, or equivalently $b_i \geq \widehat{b}_i^k$. The undominated strategy assumption also implies that for any given bid b, a truthful (resp. strategic) firm i must choose a project $f \in f_i^T(b)$ (a pair of project and declaration $(f, \sigma) \in Q_i^*(b)$). Moreover, for a given equilibrium bid b_i, we assume that a strategic firm will prefer a pair (f_i, σ_i) such that $\sigma_i = \tilde{\sigma}(f_i)$ if there is such a pair in $Q_i^*(b)$: if being truthful does not reduce its payoff, then a strategic firm will prefer to make a truthful declaration.[12]

An aspect of the procurement game that is central to our analysis is whether firms would benefit from deviating from a truthful declaration about their project, i.e., whether the optimal decision d_i of any given strategic firm i is such that $\sigma_i \neq \tilde{\sigma}(f_i)$.

Definition 2. *For a given bid b and a given firm i (characterized by the cost function C_i and the utility function U_i), we say that the procurement/menu of contracts is strategy-proof (resp. manipulable) if $\Pi_i^S(b) = \Pi_i^T(b)$ (resp. $\Pi_i^S(b) > \Pi_i^T(b)$). We say that a procurement/menu of contracts is strongly strategy-proof if it is strategy-proof for any given bid b, cost function C_i and utility function U_i.*

Let $f \in f_i^T(b)$ be an optimal project for a truthful firm i with bid b. According to the definition above, if the procurement is strategy-proof for bid b and firm i, then the same project and report are also optimal for a strategic firm, i.e., $(f, \tilde{\sigma}(f)) \in Q_i^*(b)$. Since we have made the assumption that a strategic firm will prefer a truthful decision if it does not reduce its payoff, we have that in a strongly strategy-proof procurement every strategic firm i will declare its project truthfully: $\sigma_i = \tilde{\sigma}(f_i)$. Conversely, if the procurement is manipulable for a given bid b and firm i, then for any pair $(f, \sigma) \in Q_i^*(b)$ we have $\tilde{\sigma}(f) \neq \sigma$, i.e., any pair (f, σ) that is optimal for a strategic firm i implies a false declaration.

Definition 3. *For a given contractor having won with a decision $d = (b, f, \sigma)$, we say that the buyer suffers from deception if her expected payoff $v \cdot \bar{q}_f - \mathbb{E}_f[T_{(b,\sigma)}(q)]$ is strictly smaller than $\min_{f' \in \mathcal{F} | \tilde{\sigma}(f') = \sigma} v \cdot \bar{q}_{f'} - \mathbb{E}_{f'}[T_{(b,\sigma)}(q)]$.*

In words, the buyer suffers from deception if her payoff is strictly smaller than the lowest possible payoff she could expect with a truthful contractor.[13] Henceforth, if the contractor is

[11] Standard refinements, such as the trembling-hand perfect equilibrium (see Fudenberg and Tirole, 1991), allow those irrelevant equilibria to be eliminated.

[12] This is in line with Chen, Kartik and Sobel's (2008) selection criterion in cheap talks.

[13] Note that the buyer is agnostic about the portfolios of projects $\{\mathcal{F}_i\}_{i=1,\dots,N}$ for the firms: she implicitly considers that any project $f \in \mathcal{F}$ could be chosen by the contractor, and then deception is not related to the contractor's identity.

truthful, the buyer cannot suffer from deception. If the procurement is strongly strategy-proof, then the contractor always behaves as a truthful firm (even if it is strategic). We then obtain:

Corollary 2. *If the procurement is strongly strategy-proof, then the buyer never suffers from deception.*

Under a menu of contracts where $T_{(b,\sigma)}$ does not depend on σ, it is straightforward that the procurement is strongly strategy-proof. In equilibrium, the fact that firm i wins the procurement with bid b, for which the procurement is manipulable, does not imply that the buyer suffers from deception. In fact, without further restrictions, the contractor may choose a pair (f, σ) such that $v \cdot \bar{q}_f - \mathbb{E}_f[T_{(b,\sigma)}(q)] > \min_{f' \in \mathcal{F} | \tilde{\sigma}(f') = \sigma} v \cdot \bar{q}_{f'} - \mathbb{E}_{f'}[T_{(b,\sigma)}(q)]$, i.e., the buyer may get a payoff that is larger than the smallest possible payoff she could expect with a truthful contractor. It will become clear later why, in the class of procurement games of interest, failure of strategy-proofness then leads to deception.

3.1 Production-insuring menus of contracts

Given our partial order comparing contracts with respect to insurance provision, and our class of procurement games – or equivalently, menus of contracts – for a given partition Σ of $\mathcal{F}$, we obtain a partial order comparing procurement games with respect to insurance provision.

Definition 4. *Consider two menus of contracts $\mathcal{M} = \{T_{(b,\sigma)}\}_{(b,\sigma) \in \mathbb{R} \times \Sigma}$ and $\mathcal{M}' = \{T'_{(b,\sigma)}\}_{(b,\sigma) \in \mathbb{R} \times \Sigma}$. We say that $\mathcal{M}'$ provides more insurance than $\mathcal{M}$ if $T'_{(b,\sigma)}$ provides more insurance than $T_{(b,\sigma)}$ on σ for any pair $(b, \sigma) \in \mathbb{R} \times \Sigma$.*[14]

If a menu of contracts provides more insurance than another, then for any given decision made by a truthful contractor, his payoff is greater in the former than in the latter. Increasing the expected payoff of risk-averse truthful firms for any given bid then leads to a decrease in their zero-profit bid, as formalized by the first part of the following proposition.

Proposition 3. *If a menu of contracts $\mathcal{M}'$ provides more insurance than a menu $\mathcal{M}$, then the zero-profit bid of any truthful firm i is lower under $\mathcal{M}'$ than under $\mathcal{M}$, and strictly lower if i is strictly risk-averse. Formally, $\widehat{b}_i^T(\mathcal{M}') \leq \widehat{b}_i^T(\mathcal{M})$ if U_i is concave, and the inequality is strict if U_i is strictly concave.*

Furthermore, if $\mathcal{M}$ is strongly strategy-proof, then we have $\widehat{b}_i^S(\mathcal{M}') \leq \widehat{b}_i^S(\mathcal{M})$ if U_i is concave, and the inequality is strict if U_i is strictly concave.

[14] We assume here implicitly that for any pair $(b, \sigma) \in \mathbb{R} \times \Sigma$, $\mathbb{E}_f[T_{(b,\sigma)}(q)] = \mathbb{E}_f[T'_{(b,\sigma)}(q)]$ for any $f \in \sigma \equiv \{f \in \mathcal{F} | \tilde{\sigma}(f) = \sigma\}$.

Proposition 14 in the SA characterizes the equilibrium under full complete information, i.e., when both the firms' technological and payoff characteristics $(\mathcal{F}_i, U_i, C_i)$, $i = 1, \ldots, N$, and their strategic/truthful types are common knowledge: in equilibrium, the winning bid is equal to the second-lowest zero-profit bid among the firms. As a corollary of Proposition 3, we obtain results on how the equilibrium bid submitted by the contractor is lowered when the buyer switches to a menu that provides more insurance.

Corollary 4. *Assume complete information and consider a menu of contracts $\mathcal{M}'$ providing more insurance than the menu $\mathcal{M}$. If either all firms are truthful or the menu $\mathcal{M}$ is strongly strategy-proof, then the equilibrium bid is lower under $\mathcal{M}'$ than under $\mathcal{M}$.*[15]

Corollary 4 should be interpreted with caution, because a lower winning bid does not imply that the buyer is better off. On the one hand, the buyer's payoff typically depends on the project chosen by the contractor, and that project may change if the contract is modified. Insurance could reduce firms' aversion to riskier projects, thereby inducing a project choice that is more in line with the risk-neutral buyer's preference. However, lower bids also reduce the incentive to select projects with high expected production, as discussed later in Section 4.1. This corresponds to the moral hazard issue that has already received attention in the literature. In addition, the project may change because the contractor (the identity of the most competitive firm) changes, which corresponds to the adverse selection issue. Note, however, that these channels (which do not depend on the presence of strategic firms) do not play any role if the sets of optimal projects $f_i^T(b)$ are singletons and do not depend on i or b.[16] On the other hand, the buyer's expected payoff also depends on the contractor's declaration σ. If the contractor is strategic, then the buyer's expected payoff could be lower under $\mathcal{M}'$ than under $\mathcal{M}$. This is the novel channel we are interested in.

4 Insurance provision based on expected production

Analyzing how providing more insurance opens the door to bid manipulation, and then possibly to buyer deception requires imposing more structure: In the following, we consider contracts and menus of contracts that provide more insurance than a linear benchmark, based on a partition according to expected production.

We call linear all contracts T such that $T(q) = \lambda \cdot q + \mu$ for any $q \in \mathbb{R}_+$ with $\mu \in \mathbb{R}$ and $\lambda \in \mathbb{R}_+^*$ with the convention that $\mathbb{R}_+^* = \mathbb{R}_+ \setminus \{0\}$. We refer to each specific one of them as the

[15] If both menus are manipulable, then we can not rank the zero-profit bids of strategic firms precluding any general result regarding equilibrium bids.

[16] We only need that $f_i^T(b)$ does not depend on b or i over the set of relevant bids (those that could be equilibrium bids) and relevant firms (those that could become the contractor in equilibrium).

(λ, μ)-linear contract. For any given set $\mathcal{F}$, the partition based on the expected production is such that $\tilde{\sigma}(f) = \tilde{\sigma}(f')$ if and only if $\bar{q}_f = \bar{q}_{f'}$, for any pair $(f, f') \in \mathcal{F}^2$. Formally, we will study menus of contracts $\mathcal{M}' := \{T_{(b,q_0)}\}_{(b,q_0)\in\mathbb{R}\times\mathbb{R}_+^*}$ that provide more insurance than a given menu $\mathcal{M} := \{T_b\}_{b\in\mathbb{R}}$ where T_b is the $(\lambda(b), \mu(b))$-linear contract.[17] In the following, such a menu $\mathcal{M}'$ will be referred to (for short) as *production-insuring* and the declaration q_0 as the *reference production*. It is straightforward that the linear menu $\mathcal{M}$ is strategy-proof, since the payoff of the contractor does not depend on any declaration. On the contrary, under the menu $\mathcal{M}'$, a strategic firm may benefit from reporting a reference production q_0 that differs from its expected production $\bar{q}_f$ (given its project choice f). If a firm reports $q_0 > \bar{q}_f$ (resp. $q_0 < \bar{q}_f$), then we say that it overstates (resp. understates) its production.

Linear contracts are commonly used and are thus, from a positive perspective, a natural benchmark that we want to improve upon when designing production-insuring contracts. In particular, our class of linear menus include both the cash auctions, if $\lambda(b)$ is a constant and $\mu(b) = b$, and the royalty auctions, if $\lambda(b) = b$ and $\mu(b)$ is a constant.[18] A procurement game in which each firm i submits a bid b_i, the firm i^w with the lowest bid wins the auction and the (λ, b_{i^w})-linear (resp. (b_{i^w}, μ)-linear) contract applies is hereafter referred to as linear cash (resp. linear royalty) procurement.

In the next subsection, we argue that linear menus of contracts are also a natural benchmark from a normative perspective, because the linear cash procurement with $\lambda = v$ (v the social value per unit of output) satisfies some general optimality properties when bidders are risk neutral. The optimality of this procurement, hereafter referred to as the Linear Cash Marginal Rewards (LCMR) procurement, follows intuitively from the fact that it aligns the contractor's interests with those of the buyer.

4.1 Optimal procurement under risk neutrality

This subsection is not intended to be a novel contribution, but rather wraps up and adapts to our setting some insights from the theory of regulation and the theory of auctions when both the buyer and the firms are risk neutral as assumed throughout this section.[19] The objective is to provide a theoretical status for our subsequent approach, using linear contracts as a benchmark to improve upon. We also assume full complete information throughout this section. Nevertheless, there is an analog of our results under incomplete information if we consider second-price mechanisms instead of a pay-as-bid procurement. Various technical details of the analysis below are relegated

[17] Given the monotonicity assumptions we make throughout our analysis about the menus of contracts, the functions $b \mapsto \lambda(b)$ and $b \mapsto \mu(b)$ are necessarily non-decreasing.

[18] We follow here the terminology used in the contingent auction literature (see Skrzypacz (2013) for a survey).

[19] See Armstrong and Sappington (2007) and Milgrom (2004) for introductions to the theory of regulation and on auctions, respectively.

to the SA.

Under the (λ, μ)-linear contract, the buyer's expected payoff reduces to $(v - \lambda) \cdot \bar{q}_f - \mu$. Furthermore, under the (v, μ)-linear contract, it is equal to the social welfare up to the additive constant μ. If a given firm i submits a bid and a reference production that induces a (v, μ)-linear contract, then it necessarily chooses a project in $\mathcal{F}_i^*$. In the LCMR procurement under complete information, in equilibrium, the contractor is then a socially optimal firm i^*, his equilibrium bid is equal to $-SW^*_{-i^*}$, and his expected payoff is $SW^* - SW^*_{-i^*} \geq 0$, i.e. equal to his contribution to welfare. The buyer's payoff is equal to $SW^*_{-i^*}$, i.e., the social welfare if i^* were excluded from the procurement. This design provides *marginal rewards* to the contractor, which is the key ingredient to guarantee social optimality as analyzed thoroughly by Hatfield et al. (2018).

Overall, the LCMR procurement implements the socially optimal allocation in our framework, which involves both moral hazard and adverse selection. When firms are payoff-symmetric, the LCMR procurement leaves no rents to the contractor and thus maximizes also the buyer's payoff, which is then SW^*. Nevertheless, the optimality of the LCMR procurement in terms of the buyer's expected payoff holds more generally with asymmetric firms if we augment the game with a costly entry stage as in Levin and Smith (1994). Regarding the timing of the game presented above, it would consist in adding an uncoordinated entry stage between stage 1 and stage 2. Similarly to Jehiel and Lamy's (2015) setting with ex ante asymmetric bidders, and as detailed in the SA (given that Jehiel and Lamy's (2015) setting does not involve moral hazard), the LCMR procurement maximizes the buyer's expected cost among all possible procurement games.

Departing from marginal rewards is known to open the door to social inefficiencies, either in the form of moral hazard (Laffont and Tirole (1986) and McAfee and McMillan (1987)) or adverse selection (Che and Kim, 2010). If a socially optimal firm i^* submits a given pair of bid and declaration that induces a (λ, μ)-linear contract with $\lambda > v$ (resp. $\lambda < v$), then it necessarily chooses a project whose expected production is (weakly) lower (resp. higher) than that of the socially optimal project f^*.[20] Furthermore, another source of inefficiency is that there is no guarantee that a socially efficient firm will win the procurement. However, small departures from marginal rewards are expected to lead to small inefficiencies. For example, if we consider a linear royalty procurement setting a unit price of $\lambda = b^w$, then the social inefficiency (i.e., the difference between the equilibrium social welfare and the first best SW^*) is bounded above by $|v - b^w| \cdot K$, where $K := \max_{i \in \{1,\ldots,N\}, f \in \mathcal{F}_i} \bar{q}_f - \min_{i \in \{1,\ldots,N\}, f \in \mathcal{F}_i} \bar{q}_f$. In words, linear royalty procurement is approximately efficient if the equilibrium per-unit payment – the winning bid b^w – is set close to

[20]Formally, consider two projects f and f', if firm i (weakly) prefers f to f' under a (λ, μ)-linear contract (i.e., if $\lambda \cdot (\bar{q}_f - \bar{q}_{f'}) \geq C_i(f) - C_i(f'))$, then it will strictly prefer f to f' under the (λ', μ')-linear contract either if $\lambda' > \lambda$ and $\bar{q}_f > \bar{q}_{f'}$ or if $\lambda' < \lambda$ and $\bar{q}_f < \bar{q}_{f'}$. We conclude by noting that the socially optimal project is the most preferred project for firm i^* under the (v, μ)-linear contract.

the per-unit social value v.[21]

Let us summarize our insights in an informal way (see the SA for formal details):

Proposition 5. *Assume risk neutrality and complete information. The LCMR procurement is socially optimal. In a linear royalty procurement, the equilibrium allocation is approximately socially optimal if the equilibrium winning bid is close to v. When firms are payoff-symmetric or if the set of bidding firms results from an uncoordinated costly entry process, the LCMR procurement also maximizes the buyer's expected payoff.*

4.2 Characterization of production-insuring contracts

For a given distribution f such that $\mathbb{E}_f[T(q)] = \mathbb{E}_f[\lambda q + \mu]$, the inequality $\mathbb{E}_f[U(T(q))] \geq \mathbb{E}_f[U(\lambda q + \mu)]$ for any concave function U corresponds to saying that the distribution of $T(q)$ dominates the distribution of $\lambda q + \mu$ according to second-order stochastic dominance (according to the definition in Mas-Colell, Whinston and Green (1995)). When characterizing the set of contracts that provide more insurance than the linear contract $\lambda q + \mu$ on a given set of distribution $\widehat{\mathcal{F}}$, we are thus interested in characterizing the contracts T such that the distribution of $T(q)$ dominates the distribution of $\lambda q + \mu$ according to second-order stochastic dominance for any distribution in $\widehat{\mathcal{F}}$.

Whether or not production-insuring contracts exist and, if so, what form they take, depends crucially on the set of distributions $\mathcal{F}$ over which we want them to provide more insurance than the linear contract. First, we take for $\mathcal{F}$ the full set of atomless distributions whose support is a compact subset of $\mathbb{R}_+$, i.e. the set $\mathcal{F}_{all}$. We then denote $\mathcal{F}_{all}(q_0) := \{f \in \mathcal{F}_{all} \mid \bar{q}_f = q_0\}$, for any reference production $q_0 > 0$. We show that the set of contracts that provide more insurance than a given (λ, μ)-linear contract on $\mathcal{F}_{all}(q_0)$ corresponds to the linear contracts that intersect the (λ, μ)-linear contract at q_0 but are less steep, i.e., have a slope inferior to λ. Second, we take for $\mathcal{F}$ the symmetric and single-peaked distributions in $\mathcal{F}_{all}$, i.e. the set $\mathcal{F}_{sym}$. We then denote $\mathcal{F}_{sym}(q_0) := \{f \in \mathcal{F}_{sym} \mid \bar{q}_f = q_0\}$, for any reference production $q_0 > 0$. We characterize the set of contracts that provide more insurance than a given linear contract on $\mathcal{F}_{sym}(q_0)$.

Proposition 6. *A contract T provides more insurance than the (λ, μ)-linear contract (with $\lambda > 0$) on $\mathcal{F}_{all}(q_0)$ if and only if it is a (λ', μ')-linear contract with $0 < \lambda' < \lambda$ and $\mu' = \mu + (\lambda - \lambda') \cdot q_0$.*

The main steps in the proof of Proposition 6 are as follows: First, the mean-preserving condition that $\mathbb{E}_f[T(q)] = \lambda \cdot q_0 + \mu$ for any $f \in \mathcal{F}_{all}(q_0)$, implies that the contract T must be linear and that $T(q_0) = \lambda \cdot q_0 + \mu$. Conversely, any (λ', μ')-linear contract with $\mu' - \mu = (\lambda - \lambda') \cdot q_0$ satisfies

[21] In contrast, large inefficiencies can arise if the difference $|v - b^w|$ is large, which can occur when firms with the lowest zero-profit bids are those that have very low costs but also provide very low social benefits (Che and Kim, 2010).

the mean-preserving condition. Second, from showing that the expected utility $\mathbb{E}_f[U(\lambda' \cdot q + \mu')]$ is decreasing in λ' (thanks to alternative characterizations of second-order stochastic dominance), we obtain that production-insuring contracts must be flatter ($\lambda' < \lambda$) to be strictly preferred under risk aversion.

From Proposition 6 for such a production-insuring contract T we obtain that:

$$T(q) - [\lambda \cdot q + \mu] = \underbrace{(\lambda - \lambda')}_{>0} \cdot (q_0 - q) \tag{4}$$

for any realization q. By taking the expectation, we then obtain as a corollary:

Corollary 7. *Consider a contract T that provides more insurance than the (λ, μ)-linear contract on $\mathcal{F}_{all}(q_0)$ and take $f \in \mathcal{F}_{all}$ the actual production distribution. If $\bar{q}_f < q_0$ (resp. $\bar{q}_f > q_0$), then the expected payment by the buyer to the contractor is strictly greater (resp. smaller) under T than under the (λ, μ)-linear contract.*

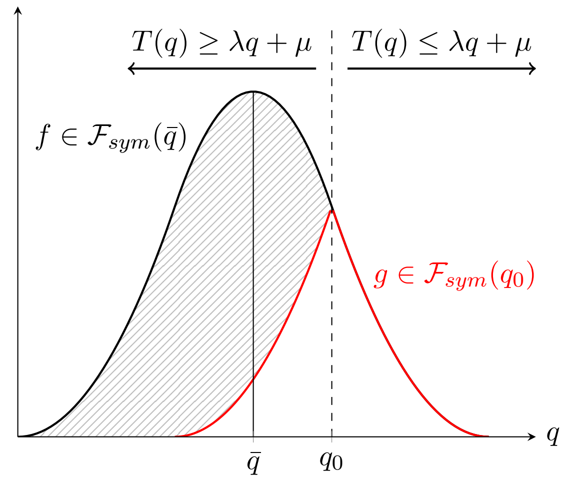
The requirement to be robust with a completely prior-free approach among the set of distributions with a given expectation leads to a very narrow set of contracts with the unpleasant property that the compensation (the difference in payment from the benchmark contract) increase with $q_0 - q$, as reflected in (4): this further implies that the contractor has an obvious incentive to overstate his expected production. One way to escape this problem might be to impose some constraints on the portfolio of possible projects $\mathcal{F}$, i.e. to consider a subset of $\mathcal{F}_{all}$. In fact, restricting $\mathcal{F}$ to symmetric and single-peaked distributions greatly expands the set of production-insuring contracts, and provides a micro-foundation for a much more relevant class of contracts.

Proposition 8. *A contract T provides more insurance than the (λ, μ)-linear contract (with $\lambda > 0$) on $\mathcal{F}_{sym}(q_0)$ if and only if:*

1. *$T(q) \geq \lambda \cdot q + \mu$ for any $q \in [0, q_0]$,*
2. *$\frac{1}{2} \cdot T(q_0(1-\epsilon)) + \frac{1}{2} \cdot T(q_0(1+\epsilon)) = \lambda \cdot q_0 + \mu$, for any $\epsilon \in [0, 1]$, and*
3. *$\int_{q_0(1-\epsilon)}^{q_0} [T(q) - (\lambda q + \mu)] dq > 0$ for any $\epsilon \in (0, 1]$.*

Condition 1 says that a contract that provides more insurance than the (λ, μ)-linear contract would never deflate payments for production realizations that are lower than the reference production q_0. In addition, the contract should satisfy a symmetry property (condition 2) reflecting that bad and good outcomes should balance each other out so that the contract provides the same expected revenue as the benchmark: $\mathbb{E}_f[T(q)] = \lambda \cdot q_0 + \mu$ for any $f \in \mathcal{F}_{sym}(q_0)$. Note that this symmetry condition, combined with condition 1, imposes that $T(q) \leq \lambda \cdot q + \mu$ for any

Figure 2: Proof of Proposition 9 – Illustration



$q \in [q_0, 2q_0]$.[22] Last, condition 3 imposes that the contract should strictly differ from the (λ, μ)-linear contract in the left neighborhood of the reference production q_0, so that the contract that provides more insurance is strictly preferred by a strictly risk-averse firm, no matter how small the support of its production distribution.

The next result is the analog of Corollary 7, and is a fundamental step in our analysis of strategic misreporting (Section 4.3). It analyzes the effect on the buyer's expected payment to the contractor when the contract T provides more insurance than a given linear contract on the set $\mathcal{F}_{sym}(q_0)$, and when the actual expected production $\bar{q}$ differs from the reference production q_0.

Proposition 9. *Consider a contract T that provides more insurance than the (λ, μ)-linear contract on $\mathcal{F}_{sym}(q_0)$ and take $f \in \mathcal{F}_{sym}$ the actual production distribution. If $\bar{q}_f < q_0$, the buyer's expected payment to the contractor is greater under T than under the (λ, μ)-linear contract. Formally, $\mathbb{E}_f[T(q)] \geq \lambda \cdot \bar{q} + \mu$ and the inequality is strict if we also have $f'(q_0) < 0$. Conversely, if $\bar{q}_f > q_0$ and $T(q) \leq \lambda \cdot q + \mu$ for $q \geq 2 \cdot q_0$, then the buyer's expected payment to the contractor is smaller under T than under the (λ, μ)-linear contract. Formally, $\mathbb{E}_f[T(q)] \leq \lambda \cdot \bar{q} + \mu$ and the inequality is strict if we also have $f'(q_0) > 0$.*

Unlike Corollary 7, the proof of Proposition 9 does not result from simple calculus. It relies on the linear decomposition of the distribution f into two other distributions, denoted by g and h, such that $f = \alpha \cdot g + (1 - \alpha) \cdot h$ with $\alpha \in [0, 1]$, and where $g \in \mathcal{F}_{sym}(q_0)$. This decomposition

[22]But, it imposes nothing about payment for realizations greater than twice the reference production, as it should never occur under truthful reporting.

is illustrated in Figure 2, where the shaded area corresponds to the mass $1-\alpha$ and the white area below the density f corresponds to the realizations from the distribution g. The fact that g is a symmetric distribution centered on q_0 implies that the expected payment with contract T under the distribution g is exactly equal to the expected payment under the (λ,μ)-linear contract (i.e., $\mathbb{E}_g[T(q)] = \lambda \cdot \mathbb{E}_g[q] + \mu$). The second distribution h has a support that is entirely below (resp. above) q_0 if $\bar{q} < q_0$ (resp. $\bar{q} > q_0$) which implies that the payment derived from T under the distribution h is greater (resp. smaller) than the payment obtained under the linear contract.

4.3 Misreporting and its consequence for the buyer

We now consider production-insuring menus and analyze the equilibrium behavior of strategic firms faced with them. For both $\mathcal{F} = \mathcal{F}_{all}$ and $\mathcal{F} = \mathcal{F}_{sym}$, and when risk aversion is limited, we establish that firms benefit from overstating their expected production and that it is detrimental to the buyer.

Consider a menu $\mathcal{M} := \{T_{(b,q_0)}\}_{(b,q_0)\in\mathbb{R}\times\mathbb{R}_+^*}$ that is production-insuring on $\mathcal{F}_{all}$ against the menu of linear contracts $\{\lambda(b)\cdot q + \mu(b)\}_{b\in\mathbb{R}}$. From Proposition 6, there exists a function $\lambda^*(b,q_0)$ such that $T_{(b,q_0)}(q) = \lambda^*(b,q_0)\cdot q + \mu(b) + (\lambda(b) - \lambda^*(b,q_0))\cdot q_0$ with $0 < \lambda^*(b,q_0) < \lambda(b)$ for any pair (b,q_0). If a risk-neutral firm chooses project $f \in \mathcal{F}_{all}$, then the difference between its expected payoff from reporting q_0 and from being truthful is $(\lambda(b) - \lambda^*(b,q_0))\cdot(q_0 - \bar{q}_f)$, which implies that overstating (resp. understating) its production increases (resp. decreases) its expected payoff. In equilibrium, the optimal decision (b,f,q_0) of a strategic risk neutral firm involves overstating production, i.e., we must have $q_0 > \bar{q}_f$, in which case the buyer's expected payoff is strictly smaller than $(v - \lambda(b))\cdot\bar{q}_f - \mu(b)$. If we make the additional assumption $\lambda(b) < v$, reflecting that both the buyer and the contractor benefit from higher realizations under the linear contract of reference,[23] then the buyer's expected payoff with a strategic contractor is less than than it would have been had the contractor chosen a project in $\mathcal{F}_{all}(q_0)$: this implies that the buyer always suffers from deception in equilibrium when the contractor is strategic. These insights also hold for production-insuring menus on $\mathcal{F}_{sym}$. The next Theorem summarizes these insights (the more formal version, which makes some technical assumptions explicit, is relegated to the Appendix).

Theorem 10. *Consider a menu $\mathcal{M} := \{T_{(b,q_0)}\}_{(b,q_0)\in\mathbb{R}\times\mathbb{R}_+^*}$ that is production-insuring relative to the menu of linear contracts $\{\lambda(b)\cdot q + \mu(b)\}_{b\in\mathbb{R}_+^*}$ either on $\mathcal{F} = \mathcal{F}_{all}$ or on $\mathcal{F} = \mathcal{F}_{sym}$. In the later case, assume also that $T_{(b,q_0)}(q) \le \lambda(b)\cdot q + \mu(b)$ for any pair (b,q_0) with $q > 2\cdot q_0$. In equilibrium, if firms are risk neutral (or if firms' risk aversion is small enough), strategic firms overstate their*

[23]If $\lambda(b) > v$, then a malicious buyer should sabotage the contractor's project. The assumption $\lambda(b) < v$ for relevant bids (i.e., formally, for bids below zero-profit bids) can thus be interpreted as a natural robustness requirement. An alternative way to guarantee that we always have $\lambda(b) < v$ for the winning bid would be to introduce a reserve price above which bids are never accepted.

production. If the contractor is a strategic firm whose bid b satisfies $\lambda(b) \leq v$, the buyer suffers from deception.

Theorem 10 formalizes a fundamental conflict between insurance and strategy-proofness. We stress that firms' incentives to overestimate their expected production hold for any distribution in $\mathcal{F}_{all}$ if $\mathcal{F} = \mathcal{F}_{all}$ (resp. $\mathcal{F}_{sym}$ if $\mathcal{F} = \mathcal{F}_{sym}$). This general result holds if the contractor is risk neutral or, by continuity of the maximum (Berge, 1963), if risk aversion is low enough. However, higher levels of risk aversion may alter (mis)reporting incentives and induce firms to understate their expected production: in particular (depending on the design of the contract) understating may be a way of hedging against the worst production outcomes, which may weigh heavily on the expected utility of a very risk-averse firm.[24] From an practical perspective, this channel may not play a significant role. Nevertheless it prevents us from deriving the overstating insight of Theorem 10 for any utility function.

Misreporting intensity Theorem 10 is silent on the magnitude of misreporting, which can intuitively be viewed as a proxy for the flaws resulting from the presence of strategic bidders. Let us analyze the reference production that maximizes the contractor's expected payoff for a given bid b and a given project $f \in \mathcal{F}_{sym}$. To gain further insight, we impose more structure on our model.

We consider a class of menus of contracts (parameterized by $w \in [0, 1]$) that are production-insuring relative to the (λ, μ)-linear contract, where for any $q_0 > 0$ the payment is completely flat within a range of $\pm w\%$ around the reference production q_0, and matches the (λ, μ)-linear contract outside this range.[25] In addition, we assume that the insurance range is large enough to fully insure the contractor under truthful reporting (i.e., w is large enough such that the support of f is included in $[(1-w)\bar{q}, (1+w)\bar{q}]$), and that the PDF f is continuous on $\mathbb{R}_+$, and such that $x \mapsto \frac{1-F(x)}{f(x)}$ is decreasing on the interior of its support.

Under such assumptions, we obtain the following results (the proofs of which are detailed in the SA): any optimal reference production of a risk-averse (strategic) contractor is above the true expected production $\bar{q}$ and below the optimal report of a risk-neutral contractor (the latter does not depend on (λ, μ). With the additional restriction that the function $q \mapsto U_i(\lambda q + \mu - C_i(f))$ is a CRRA utility function, i.e., if we take $U_i(x) := \frac{[\frac{x+C_i(f)-\mu}{\lambda}]^{1-\gamma}}{1-\gamma}$ $(\gamma \geq 0)$, we obtain that the set of optimal reports $\operatorname{Arg\,max}_{q_0>0} \{\Pi_i(b, f, q_0)\}$ is a singleton. We derive the following comparative statics about the corresponding optimal report denoted $q^*_{w,f,\gamma}$:

[24] If one considers a menu of contracts similar to the one shown in the right panel of Figure 1, understating is a way to ensure that the lowest realizations fall in the part of T that provides insurance relative to the linear contract (i.e. where $T(q) > \lambda \cdot q + \mu$), and not in a part where it is equivalent to the linear contract.

[25] For the sake of technical simplicity, we have assumed throughout our previous analysis that the production contracts are (strictly) increasing. This difference is actually innocuous.

1. The lower is the coefficient of relative risk aversion γ, the higher is $q^*_{w,f,\gamma}$.
2. Considering two production distributions F_1 and F_2, where F_1 is less risky than F_2 in the sense that $\frac{f_1(q)}{(1-F_1(q))} \leq \frac{f_2(q)}{(1-F_2(q))}$ for any $q \leq \bar{q}$, then the optimal report $q^*_{w,f,\gamma}$ is higher when the contractor faces the least risky distribution F_1 than when he faces the most risky distribution F_2.
3. The larger is the insurance range w, the higher is $q^*_{w,f,\gamma}$ if $\gamma \geq 1$.

5 Analysis of the buyer's expected cost without moral hazard

In this section, we characterize how the manipulability of production-insuring contracts affect the buyer's expected cost in equilibrium. In order to get clear-cut results we abstract from the fact that the contractor's project may change when the menu of contracts changes, and thus focus on the potential misallocation of the contract and the rents captured by the winning firm.

Formally, we consider environments where all firms have access to a unique common project f ($\mathcal{F}_i = \{f\}$ for each i), and we use the notation $c_i \equiv C_i(f)$ for firm i's cost. We also assume that firms have the same utility function U. We are still considering a pay-as-bid lowest-price auction. Under such circumstances, maximizing the buyer's expected payoff is equivalent to minimizing the buyer's expected cost. We analyze the consequences of switching from a given strongly strategy-proof menu of contracts $\mathcal{M} := \{T_{(b,\sigma)}\}_{(b,\sigma)\in\mathbb{R}\times\Sigma}$ to a menu of contracts $\mathcal{M}' := \{T'_{(b,\sigma)}\}_{(b,\sigma)\in\mathbb{R}\times\Sigma}$ that provides more insurance than $\mathcal{M}$.

Our subsequent analysis relies on the assumption that the production-insuring menu $\mathcal{M}'$ is manipulable for any firm and any bid. As formalized in Theorem 10, this is verified in particular when we take $\mathcal{F}_{sym}$ for the portfolio of possible projects $\mathcal{F}$, the expectation-based partition for Σ, a linear menu of contracts $\{\lambda(b) \cdot q + \mu(b)\}_{b\in\mathbb{R}}$ for $\mathcal{M}$ and when firms are risk neutral or little risk averse.

We first consider a setting with full complete information where we allow firms to have asymmetric costs. We then consider a payoff-symmetric setting with incomplete information about whether each firm is strategic or truthful.

5.1 Complete information

Let us rank the costs of the N firms such that $c_1 \leq c_2 \leq \cdots \leq c_N$. A socially efficient firm is one whose cost equals c_1. Under the strongly strategy-proof menu $\mathcal{M}$, firm i's zero-profit bid, which does not depend on its truthful/strategic type and is the denoted by $\widehat{b}_i(\mathcal{M})$, is characterized by

$$\mathbb{E}_f[U(T_{(\widehat{b}_i(\mathcal{M}),\tilde{\sigma}(f))}(q) - c_i)] = 0 \tag{5}$$

which implies that $\widehat{b}_1(\mathcal{M}) \leq \cdots \leq \widehat{b}_N(\mathcal{M})$ since the function $b \mapsto T_{(b,\tilde{\sigma}(f))}(q)$ is increasing in b. We obtain that a socially efficient firm becomes the contractor and his equilibrium bid is the zero-profit bid of the second most efficient firm $\widehat{b}_2(\mathcal{M})$. From Jensen's inequality we have $\mathbb{E}_f[T_{(\widehat{b}_2(\mathcal{M}),\tilde{\sigma}(f))}(q)] \geq c_2$, where the inequality is strict if U is strictly concave and stands as an equality if U is linear. Therefore, the buyer's expected cost is equal to c_2 under risk neutrality and greater than c_2 under risk aversion.

Under the production-insuring menu $\mathcal{M}'$, a firm's zero-profit bid depends on whether it is truthful or strategic. If firm i is truthful, then its zero-profit bid $\widehat{b}_i^T(\mathcal{M}')$ is characterized in a similar way as in (5) and we have $\widehat{b}_1^T(\mathcal{M}') \leq \cdots \leq \widehat{b}_N^T(\mathcal{M}')$. If all firms are truthful, then the equilibrium bid is equal to $\widehat{b}_2^T(\mathcal{M}')$, which is smaller than $\widehat{b}_2(\mathcal{M})$ (from Proposition 3), which in turn implies that the buyer's expected cost would be lower with $\mathcal{M}'$ than with $\mathcal{M}$: formally, $\mathbb{E}_f[T'_{(\widehat{b}_2(\mathcal{M}'),\tilde{\sigma}(f))}(q)] \leq \mathbb{E}_f[T'_{(\widehat{b}_2(\mathcal{M}),\tilde{\sigma}(f))}(q)] = \mathbb{E}_f[T_{(\widehat{b}_2(\mathcal{M}),\tilde{\sigma}(f))}(q)]$ where the latter equation corresponds to the mean-preserving condition between $\mathcal{M}$ and $\mathcal{M}'$.

If firm i is strategic, its zero-profit bid $\widehat{b}_i^S(\mathcal{M}')$ is characterized by

$$\Pi_i^S(\widehat{b}_i^S(\mathcal{M}')) = \max_{\sigma\in\Sigma} \mathbb{E}_f[U(T_{(\widehat{b}_i^S(\mathcal{M}'),\sigma)}(q) - c_i)] = 0.$$

Again, the cost ranking translates directly into the ranking of zero-profit bids: $\widehat{b}_1^S(\mathcal{M}') \leq \cdots \leq \widehat{b}_N^S(\mathcal{M}')$. The manipulation assumption regarding the menu $\mathcal{M}'$ implies that the zero-profit bid of strategic firm is strictly smaller than that of truthful firms: $\widehat{b}_i^S(\mathcal{M}') < \widehat{b}_i^T(\mathcal{M}')$ for any firm i.

Nevertheless, lower equilibrium bids under the production-insuring menu $\mathcal{M}'$ do not translate into lower buyer's expected cost given that $\mathbb{E}_f[T'_{(\widehat{b}_2(\mathcal{M}),\sigma)}(q)]$ may be larger than $\mathbb{E}_f[T_{(\widehat{b}_2(\mathcal{M}),\tilde{\sigma}(f))}(q)]$ when $\sigma \neq \tilde{\sigma}(f)$.

Under risk neutrality, the buyer's expected cost equals the contractor's surplus minus social welfare. Switching to a menu that suffers from manipulability affects the buyer's expected cost through social welfare and through the rent captured by the winning firm. On the one hand, switching from the strongly strategy-proof menu $\mathcal{M}$ to the menu $\mathcal{M}'$ reduces social welfare if the identity of the winning firm changes, namely if a strategic inefficient firm is able to outbid the most efficient firm. On the other hand, it could change the contractor's surplus: e.g. increase it if the most efficient firm is the unique strategic firm, or decrease it if the truthful most efficient firm faces a strategic competitor. Therefore, the overall effect on the buyer's expected cost is indeterminate. Proposition 11 establishes that this depends critically on the strategic/truthful type of the firm that becomes the contractor.

Proposition 11. *Assume full complete information and that all firms are risk neutral. Under a strongly strategy-proof menu $\mathcal{M}$, the buyer's expected cost is equal to the second lowest cost c_2 and the contractor, which is a socially efficient firm, makes the surplus $c_1 - c_2$. Consider then a menu*

$\mathcal{M}'$ that is manipulable for any firm and any bid. The buyer's expected cost under $\mathcal{M}'$ depends on whether the firm that becomes the contractor is truthful or strategic:

- *If the contractor is a strategic firm, then the buyer's expected cost is (weakly) larger than c_2. The inequality is strict if the contractor is the unique strategic firm, and stands as an equality if all firms are strategic.*
- *If the contractor is a truthful firm, then the contractor is necessarily a socially efficient firm and the buyer's expected cost is (weakly) smaller than c_2. The inequality is strict if the contractor is the unique truthful firm, and stands as an equality if all firms are truthful.*

Note that whether the contractor is a strategic or truthful firm is endogenous. If the most efficient firm is strategic, then the contractor must be a socially efficient firm. If it is truthful, it may or may not be outbid by a strategic competitor, depending on their respective costs.

If firms' costs are homogeneous ($c_i = c$ for each i) in which case we drop the subscript i, then the contractor is truthful only if all firms are truthful, and the buyer's expected cost is then equal to c. Under homogeneous costs and risk neutrality, there is thus a single case where the buyer's expected cost is actually strictly greater than c. This happens when there is a single strategic bidder: the equilibrium bid under the manipulable menu $\mathcal{M}'$ is then equal to $\widehat{b}^T(\mathcal{M}')$, and the buyer's expected cost is equal to

$$c + \underbrace{\max_{\sigma \in \Sigma} \mathbb{E}_f [T_{(\widehat{b}^T(\mathcal{M}),\sigma)}(q) - T_{(\widehat{b}^T(\mathcal{M}),\tilde{\sigma}(f))}(q)]}_{:=\Delta} . \tag{6}$$

In other words, the strategic firm captures the surplus $\Delta = \Pi^S(\widehat{b}^T(\mathcal{M}')) - \Pi^T(\widehat{b}^T(\mathcal{M}')) > 0$, which is strictly positive given our manipulability assumption.

Overall, under risk neutrality, production-insuring menus are only beneficial if strategic firms are sufficiently inefficient that the comparative advantage of strategic reporting is insufficient to offset the cost disadvantage.[26]

Under risk aversion, there is a third effect at work when comparing the buyer's expected cost under $\mathcal{M}$ and under $\mathcal{M}'$: the corresponding equilibrium bids are also driven by risk premiums. As a result of insurance provision, the zero-profit bids of truthful firms are strictly lower under $\mathcal{M}'$ than under $\mathcal{M}$. The zero-profit bids of strategic firms are pushed even lower but the overall effect on the buyer's expected cost may be deceptive: a lower bid do not necessary reflect

[26] The possible pro-competitive effect of having a subset of bidders who are strategic is reminiscent of the literature on corruption, and in particular of Burguet and Che (2004), where bid manipulations could sometimes be a way to reduce the contractor's rents. In an incomplete information model of the first-price auction with favoritism, Burguet and Perry (2007) surprisingly show that the manipulation is beneficial to the buyer when the dishonest bidder is a strong bidder.

lower risk premiums in this case but rather strategic firms' ability to raise their expect payoff thanks to a false declaration $\sigma \neq \tilde{\sigma}(f)$. If the contractor is a truthful firm, a strictly lower equilibrium under $\mathcal{M}'$ than under $\mathcal{M}$ implies that the buyer's expected cost is strictly smaller under $\mathcal{M}'$ than under $\mathcal{M}$ (similarly to Proposition 11). On the contrary, the overall effect on the buyer's cost is ambiguous when the contractor is a strategic firm. For example, if firms are payoff-symmetric and there is a single strategic firm, then the equilibrium bid under the production-insuring menu is $\widehat{b}^T(\mathcal{M}')$, which is less than the equilibrium bid $\widehat{b}(\mathcal{M})$ under the menu $\mathcal{M}$ (following Proposition 3). However, the buyer's expected cost is $\mathbb{E}_f[T_{(\widehat{b}^T(\mathcal{M}'),\sigma_0^*)}(q)]$, where $\sigma_0^* \in \operatorname{Arg\,max}_{\sigma\in\Sigma} \mathbb{E}_f[U(T_{(\widehat{b}^T(\mathcal{M}'),\sigma)}(q) - c)]$, whose ranking relative to the buyer's expected cost under $\mathcal{M}$, which is equal to $\mathbb{E}_f[T_{(\widehat{b}^T(\mathcal{M}),\tilde{\sigma}(f))}(q)] > \mathbb{E}_f[T_{(\widehat{b}^T(\mathcal{M}'),\tilde{\sigma}(f))}(q)]$, is ambiguous. The ambiguity of the overall effect arises if $\mathbb{E}_f[T_{(\widehat{b}^T(\mathcal{M}'),q_0^*)}(q)] > \mathbb{E}_f[T_{(\widehat{b}^T(\mathcal{M}'),\bar{q})}(q)]$ (which holds if firms are risk-neutral and thus by continuity when risk aversion is small enough). Under risk aversion, it is thus an empirical question whether the negative consequences of misreporting dominate the benefits of insurance provision. This question is illustrated in Section 6.

5.2 Incomplete information with symmetric firms

Consider a similar setting, with homogeneous costs, but where we consider incomplete information regarding the strategic/truthful status, which is now firms' private information. When strategic/truthful types are i.i.d., we can adapt and extend Maskin and Riley's (1985) analysis of the first-price auction with binary types to characterize equilibrium in our contingent auction setting for of menu $\mathcal{M}'$ that is manipulable for any bid. In equilibrium, truthful firms bid their zero-profit bid $\widehat{b}^T(\mathcal{M}')$. On the contrary, strategic firms adopt a mixed strategy where the support of their bid distribution is $[\underline{b}, \widehat{b}^T(\mathcal{M}')]$, where the lower bound $\underline{b}$ is strictly larger than the strategic firms' zero-profit bid $\widehat{b}^S(\mathcal{M}')$. Thus, strategic firms capture some surplus. The corresponding equilibrium analysis is detailed in the SA (in a more general environment with moral hazard). Next proposition characterizes the buyer's expected cost under risk neutrality.

Proposition 12. *Assume that firms have the homogeneous cost c and are risk neutral. Suppose that each firm is strategic with probability $\alpha \in [0,1]$ independently of each other. Consider a menu $\mathcal{M}'$ that is manipulable for any bid. In equilibrium, the buyer's expected cost is equal to*

$$c + N \cdot \alpha(1-\alpha)^{N-1}\Delta \tag{7}$$

where $\Delta = \Pi^S(\widehat{b}^T(\mathcal{M}')) - \Pi^T(\widehat{b}^T(\mathcal{M}')) > 0$.

Note that $N \cdot \alpha(1-\alpha)^{N-1}$ corresponds to the probability to have a single strategic firm. This term is thus smaller than one and is actually the highest (for a given number of firms N) when

$\alpha = \frac{1}{N}$ in which case the term stands between 0.36 and 0.5.[27] We obtain thus that the buyer's expected cost is lower than in the case with a single strategic firm and that the worst case in this incomplete information setting is when $\alpha = \frac{1}{N}$ and $N = 2$ in which case the buyer's cost increase due to misreporting is divided by half compared to the case with a single strategic firm.

Under risk aversion, the rent captured by strategic firms under the production-insuring menu $\mathcal{M}$ may be offset by the benefits from the reduced risk premiums, in particular when the contractor is a truthful firms which occurs with probability $(1-\alpha)^N$. The overall effect is again ambiguous, and is an empirical question.

6 Simulations calibrated on French offshore wind procurement

In 2011 and 2013, the French government conducted auctions to allocate sites and long-term contracts for offshore wind projects with a combined capacity of approximately 3 GW. These long-term contracts, commonly referred to in the industry as *feed-in tariffs*, provided for an annual payment to the operator based on the amount of electricity produced by the wind farm and the bid placed in the auction. This bid would be approximately equivalent to the unit price of the contract, with the exception that in these auctions the French government departed from the standard linear royalty menu setting, with a payment $T_b(q) = b \cdot q$. Instead, they chose to adopt a more sophisticated menu of contracts, where the payment $T_{b,q_0}(q)$ also depends on a "reference production" q_0 reported by the candidates in the auction process. The aforementioned contracts are depicted by the solid line in Figure 3. Their precise expression is provided in SA2. Upon applying Proposition 8, it can be seen that this menu of non-linear contracts provides more insurance than the linear royalty menu $\{b \cdot q\}_{b>0}$ on the set $\mathcal{F}_{sym}$.

We calibrate our model on 5 of these offshore wind auctions, considering a setting without moral hazard and with payoff-symmetric firms. However, there are some minor differences between the calibrated model and our static theoretical framework. The contract lasts 20 years, with the contractor aiming to maximize his expected discounted profit, where costs involve both a fixed investment cost occurring before production and fixed operating costs occurring each year. The primitives of our model are the production distribution, which depends on the location and is thus auction-specific; the investment and operating costs; firms' risk aversion, which is captured through the CRRA utility function $U(x) = \frac{x^{1-\gamma}}{1-\gamma}$); and the annual discount rate. The calibration choices are described in SA2. Note that the yearly production distributions obtained from wind turbine models (Staffell and Pfenninger, 2016) are not (strictly speaking) symmetric, but the asymmetry is limited. The only parameter that is varied is the coefficient of relative risk aversion

[27] Formally, the function $x \mapsto (\frac{x}{1+x})^x$ is decreasing for $x \geq 1$, is equal to 0.5 for $x = 1$ and converges to e^{-1} when x goes to infinity.

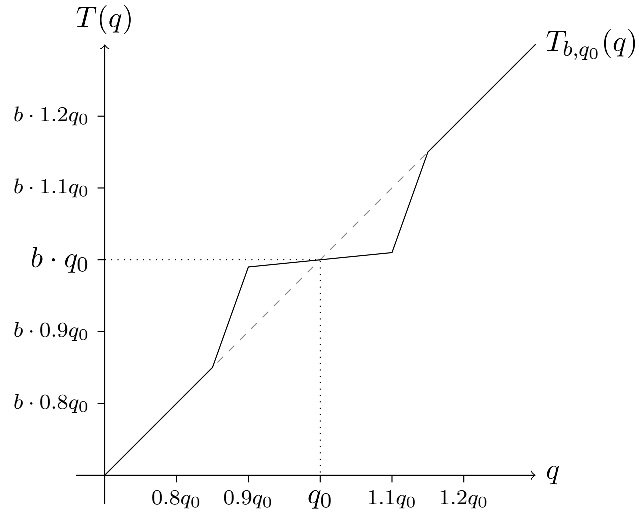


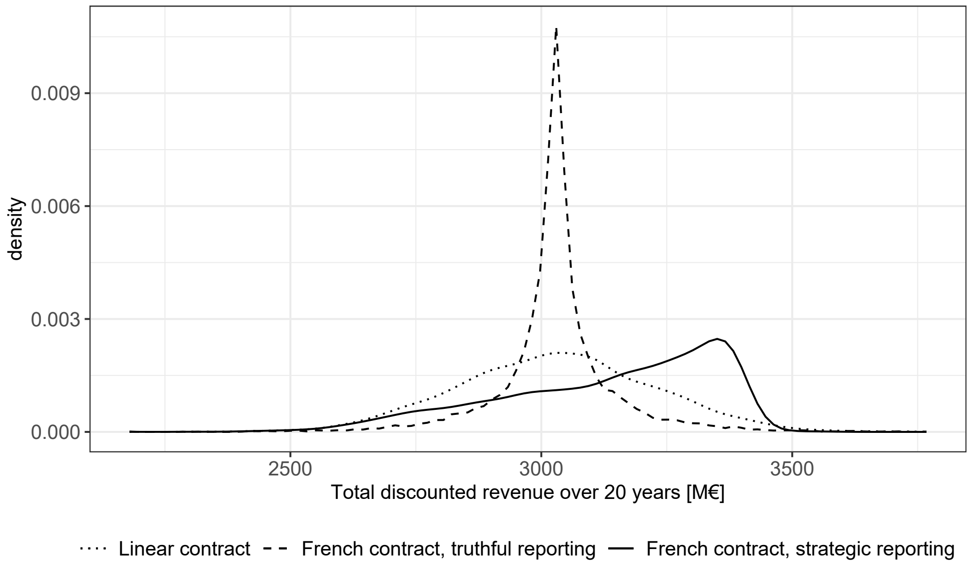
Figure 3: Contract design used in French offshore wind auctions

of the firms $\gamma \geq 0$. In the results presented hereafter, all ranges correspond to the smallest and largest results obtained among the five procurement auctions.

Figure 4 depicts the PDF of the discounted revenue raised over the duration of the contract (20 years) for the wind farm project located in Courseulles (Normandy) and the corresponding observed winning bid. Three different scenarios are considered: payments under the linear contract and under the French contract, where in the latter the contractor either truthfully reported its actual expected production, or strategically reported the reference production that maximizes its expected revenue. When firms report their expected production truthfully, we observe (as expected) that the revenue distribution is less spread out under the French contract than under the linear contract. However, by strategically overestimating their expected production, firms could benefit from a significant upward shift in their revenue distribution: we estimate that risk-neutral firms' optimal report is a reference production that is 11.9 to 12.5% greater than their actual expected production. As a result, they increase their expected revenue by 3.2 to 3.6% (compared to truthful reporting with the same price bid). Nevertheless, by doing so, they also increase the standard deviation of their revenue distribution by 72 to 85% in comparison to truthful reporting, which ends up being 10 to 13% greater than the standard deviation under the linear contract. In sum, the French contract, which was presumably designed to insure firms against production risk, may in fact increase this risk.

To pursue a comparison between the French menu of contracts and the linear royalty menu, it is also necessary to account for the effect of the contract design and firms' strategic or truthful behavior on equilibrium bids. Figure 5 depicts the buyer's expected cost in equilibrium as a

Figure 4: Distribution of the firm's discounted revenue (Courseulles wind farm)



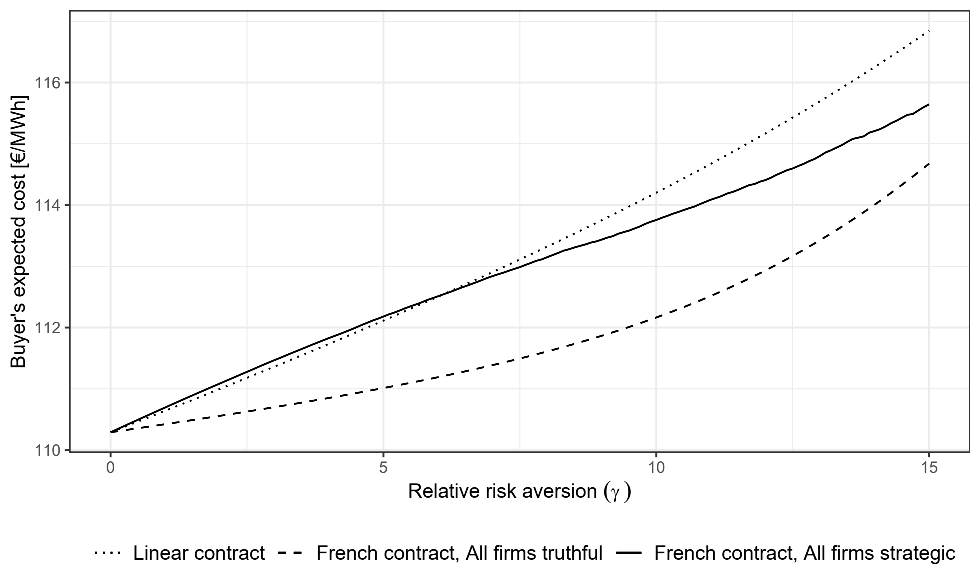
function of γ under the linear royalty menu of contracts and under the French menu of contracts, first where all firms are truthful and then where all firms are strategic. It is based on the same wind farm as Figure 4, located in Courseulles. Our estimates for the risk premiums under the linear royalty contract are notably small: for $\gamma = 1$ they are comprised between $0.29 - 0.36\%$, and fluctuate in the range $0.89 - 1.1\%$ for $\gamma = 3$. Thanks to the French contract design, this risk premium is reduced by a bit more than half when all firms are truthful. However, these (limited) gains are entirely lost when all firms are strategic and this for any reasonable level of risk aversion: for any $\gamma < 6$, the linear royalty menu actually outperforms the French production-insuring menu (see Figure 5).

Furthermore, a single strategic firm using strategic reporting to reap out a positive rent would result in the buyer's expected cost increasing by $3.3 - 3.6\%$ in comparison to the linear royalty menu when firms are risk neutral, and by $2.6 - 2.9\%$ when firms' risk aversion is up to $\gamma = 3$. For $\gamma = 1$, this increase is more than 15 times greater than the potential cost reduction in the most favorable case where all firms are truthful.

7 Discussion and extensions

Variable costs In some applications, in particular in procurement for infrastructure projects as in Bolotnyy and Vasserman (2023), ex post risk involves variable costs. Suppose that actual production q leads to the variable cost $\tilde{C}(q)$ for the contractor (with the normalization $\tilde{C}(q) = 0$),

Figure 5: Buyer's expected cost per unit of output (Courseulles wind farm)



in addition to the fixed cost associated with the project's choice. Our analysis can be adapted straightforwardly to this framework if we assume that these variable costs are observable ex post: according to our notation, it would consist in replacing the contract $T_{(b,q_0)}(q)$ by $T_{(b,q_0)}(q) + \tilde{C}(q)$. In this more general environment, the analog of the (λ, μ)-linear contracts consists first of reimbursing the observable variable costs $\tilde{C}(q)$ and then adding to this the transfer $\lambda \cdot q + \mu$. In particular, if the variable cost function $\tilde{C}$ is linear, then the analog of a linear royalty procurement remains a linear royalty procurement. From this point of view, departing from the unit price contracts commonly used in procurement for infrastructure projects – as discussed in Section 8 – to hedge against ex post risk would raise the same kind of issues.

Costly manipulation Our model can be viewed as one where the cost of falsification is binary, either zero for strategic firms or infinity for truthful firms. In practice, inflating the reference production involves typically some costs (because firms need either to produce a fake justification for it, or to corrupt the agent in charge of the technical evaluation of the project). Following Maggi and Rodriguez-Clare (1995),[28] let us briefly consider a simple model where the falsification cost is a smooth increasing function of the magnitude of the absolute difference between the reported reference production and the (true) expected production $\bar{q}$. Under risk neutrality, then it is straightforward given Theorem 10 that the optimal report with such falsification costs would lie somewhere between $\bar{q}$ and the optimal report without falsification costs. From this perspective,

[28]Costly misrepresentation has received attention in models in line with the cheap talk literature (see e.g. Kartik (2009) and Deneckere and Severinov (2022))

our results in Section 5 provide a kind of upper bound to how misreporting is prone to increase the buyer's expected cost. Nevertheless, falsification is also a wasteful activity such that increasing falsification costs is not necessary beneficial from a welfare perspective.

Alternative winner-determination rules We have considered procurement games where the winner-determination rule does not depend on firms' declarations on their project characteristics. Under a production-insuring menu based on expected production, the buyer's expected payoff associated with firm's i bid b_i and reference production q_i is equal to $(v - \lambda(b_i)) \cdot q_0 - \mu(b_i)$ if firm i is truthful. If all firms were truthful, it would thus be optimal ex post to select the winner among the set $\text{Arg}\max_{i=1,\dots,N} (v - \lambda(b_i)) \cdot q_i - \mu(b_i)$.[29] If $\lambda(b) < v$ for any possible equilibrium bid b, then it suggests that the buyer should use a winner-determination rule where the winning probability of a bidder increases with its reference production (e.g. through a scoring rule that combine both the bid and the declaration). However, it is straightforward that such rules would provide bidders an additional incentive to overstate their reference production, which would reinforce Theorem 10 and its adverse consequences.

The analysis in Section 5 is devoted to the lowest-price procurement auction. When some characteristics of a good or service are non-contractible, it is well-known that price competition can select the worst suppliers. E.g., cheap suppliers can be associated with higher risk of bankruptcy (Zheng, 2001), with time delays or with cost overruns (Decarolis, 2014). This is the reason why in many procurement, the buyer still leaves herself the choice to disqualify unreasonably low bids (Bajari, Houghton and Tadelis, 2014). More generally, departures from lowest-price auctions are popular in procurement for road construction and maintenance contracts (Decarolis, 2018): many winner-determination rules are in the vein of the average bid auction (ABA) where the winning bid is the one that is closest to the average of all the submitted bids. If there are some non-reliable firms, the idea of the ABA is to reduce the probability to select them.[30] The same problem arises in our setting as well. If firms are payoff-symmetric and if there is a single strategic firm, then the lowest-price auction select for sure the strategic firm. On the contrary, the ABA supports an equilibrium with the same equilibrium price as the lowest-price auction but where the strategic firm is selected with probability $\frac{1}{N}$. Nevertheless, the ABA is known to suffer from equilibrium multiplicity: in our setting, firms could coordinate on an equilibrium bid that is greater than the zero-profit bid of truthful firms. On the whole, departing from the lowest-price auction could mitigate the adverse selection problem but can not fully eliminate the problem when the menu of

[29] It seems here that ex post optimal auctions must rely on winner-determination rules that do not solely depend on firms' bid but also on their declarations. However, there is no conflict with the general optimality properties of the LCMR auction presented in Section 4.1 insofar as we always have $\lambda(b_i) = v$ in the LCMR auction.

[30] Relatedly, Lopomo, Persico and Villa (2023) propose of a class of mechanisms that involve both auction and lottery features and include both the lowest-price auction and the ABA as extreme cases. In their setting with adverse selection, Lopomo et al. (2023) establish that the optimal mechanism belongs to this class.

contracts is manipulable.

8 Related literature

Like Eső and White (2004), we consider an auction setting where bids incorporate risk premiums, but the connection goes no further for various reasons. Eső and White (2004) abstract from moral hazard and do not consider contingent auctions but rather analyze –and compare– standard auction formats and how informational rents interact with risk aversion. On the contrary, informational rents play a minor role in our analysis which is focused on designing risk-sharing contracts with a prior-free approach. Our work is thus mostly related to the robust mechanism design and the contingent auction literature as detailed below.

Robust mechanism design Our paper contributes to the literature on mechanism design that departs from the traditional common knowledge assumptions on the primitives of the model (e.g. on the distribution of agents' private signals). According to Carroll's (2019) classification, our analysis involves both "robustness to technology and preferences" and "robustness to distribution". Our research question is to design and analyze risk-sharing menus of contracts that reduce risk premiums for a large set of primitives including both the contractor's risk preferences and cost function and the distribution of the exogenous risk. Contrastingly, the robust mechanism design literature typically considers the maximization of the principal's expected profit[31] under the worst case over a set of possible primitives for the agent(s). For example, Garrett (2014) and Carroll and Meng (2016) revisit Laffont and Tirole's (1986) optimal procurement problem with a prior-free approach regarding the agent's effort cost function and the distribution of noise, respectively.[32]

Contingent auctions Our paper contributes to the theoretical literature on contingent auctions as surveyed by Skrzypacz (2013). In contrast to the literature supporting the VCG mechanism as mentioned earlier, a mechanism which corresponds actually to a peculiar cash-only auction in our setting, Hansen (1985) argues that royalty auctions leave lower informational rents to the winning bidder compared to cash-only auctions.[33] Abhishek, Hajek and Williams (2015) consider a model with risk-averse bidders and argue that royalties are beneficial not only because

[31] Or analogously the minimization of the principal's expected regret when Bergemann and Schlag (2008) revisit the standard monopolist pricing problem with a prior-free approach.

[32] Relatedly, Carroll (2015, 2017) considers models where the principal's knowledge on the agent's primitives suffers from partial uncertainty. Carroll (2015) departs from Bayesian approaches by considering that the principal does not know the set of technologies available to the agent but still knows that it is a superset of a known set. Carroll (2017) considers a model with multi-dimensional private signals where the principal knows the marginal distribution of each signal but adopts a prior-free approach regarding the correlation structure between signals.

[33] More generally, DeMarzo, Kremer and Skrzypacz (2005) introduce the concept of "steepness", arguing that having "steeper" securities reduces informational rents. Nevertheless, in large markets with competing sellers, Gorbenko and Malenko (2011) argue that cash-only auctions are revenue-maximizing for the seller: reducing informational rents through royalties would cost the seller through reduced participation.

they reduce informational rents but also because they provide more insurance and thus reduce risk-premiums.

The empirical literature on auctions and procurement is also taking a growing interest in auctions involving risk-sharing contracts. Bhattacharya, Ordin and Roberts (2022) estimate the optimal royalty rate in auctions for oil tract contracts through a structural model that includes drilling decisions. In procurement for transport infrastructure projects, Bolotnyy and Vasserman (2023) question the relative performance of Fixed Price (FP) contracts – where the contractor bears all the cost overruns – versus unit-price (UP) contracts that specify a percentage of the observable costs that accrue to the buyer. According to their estimates, switching to a FP contract would more than double public spending compared to a UP scaling auction where contractors are partially insured against cost overruns.

An original aspect of our analysis is that we consider that the contractor may manipulate the buyer such that risk-sharing contracts may open the door to "manipulative rents" and/or inefficiencies as in the literature on bid manipulation/gaming where some bidders may not bid according to the "spirit" of the auction rules.[34,35] The main insight from this literature is that heterogeneity between bidders' abilities or opportunities in gaming opens the door to welfare inefficiencies by selecting – instead of the firms with the lowest cost – the best "manipulators" and/or to non-competitive rents accruing to those manipulators. As an example, Ryan (2020) considers procurement auctions for coal power plants with a hedging instrument against the coal future price. The winning bidders are determined through a score combining a bid and an index of how much the firm wishes to be hedged against coal price variations. Ryan (2020) shows that some firms prefer not to use the hedging instrument in order to increase their score, having in mind their ability to renegotiate their contract in case of spikes in the price of coal.[36] In this perspective, our strategic bidders are the analog of the firms who benefit the most from ex post renegotiation in Ryan (2020).

9 Conclusion

We study procurement auctions with ex post risk. In such environments, it is tempting for the buyer to design risk-sharing contracts. We have formalized a conflict between designing robust production-

[34]It resonates with bid skewing in scaling auctions where the score of a bid is computed based on ex ante estimates but where payments depend on ex post realizations. If bidders receive, ex ante, information about actual quantities, then they will benefit from skewing their bids (Athey and Levin, 2001). In a related manner, Agarwal, Athey and Yang (2009) discuss such incentives and mention other manipulations in sponsored search auctions for online advertising.

[35]Cabrales, Calvó-Armengol and Jackson (2003) analyze opportunistic misreporting behavior in a mutual insurance scheme used in Andorra where households are invited to freely report the private monetary value they assign to their property. Some aspects of their analysis resonate with our analysis, e.g. the fact that the allocation of risk is efficient provided that households are truthful while strategic misreporting coupled with asymmetries induce inefficiencies. Nevertheless, the connection goes no further. It is notable that misreporting incentives do not always plead for over-reporting in their setting: the incentives depend on the relative wealth of the households.

[36]A related bid manipulation issue is when the procurement adjudicator is corrupted and could deliberately misevaluate some bids in exchange for a bribe. See, e.g., Celentani and Ganuza (2002), Burguet and Che (2004) and Compte, Lambert-Mogiliansky and Verdier (2005).

insuring rules and designing rules that are immune to false declarations regarding the project chosen by the contractor: in particular, strategic contractors overstate their expected production in order to increase their expected payment which induces buyer's deception. In addition, heterogeneity between bidding firm regarding their ability to misreport their expected production opens the door to a novel form of rents which accrues to strategic firms. Our analysis and the associated empirical simulations support the insight that adopting production-insuring contracts to reduce risk premiums is a risky bet without a proper screening technology preventing false declarations.

However, the class of production-insuring contracts we have analyzed rely on important restrictions. On the one hand, to discourage misreporting we may wish to consider contracts where the contractor gets punished for very low production. Given our characterisation, such punishments are not compatible with our definition of production-insuring contracts based on expected production. Our companion paper (Lamy and Leblanc, in preparation) contains a numerical investigation of a parametric class of such rules which concludes that sticking to linear contracts seems a safe choice. On the other hand, the hedging instrument is static: it does not use the fact that in some applications, the outcome can be modelled as a vector of independent draws from a common distribution. As argued in Thomas and Worrall (1990) with a repeated principal-agent setting with i.i.d. shocks, efficient risk sharing relies on dynamic contracts and repeated interactions allow asymmetric information to be reduced.[37] Relaxing these restrictions could be an interesting avenue for further research.

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[37] See Malin and Martimort (2016) and Krasikov and Lamba (2021) for more recent contributions on optimal dynamic contracts with risk aversion and cash constraints, respectively.

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Appendix: Main proofs

A Proof of Proposition 3

Case 1: truthful firms

Consider a risk-averse firm i, i.e. such that U_i is concave. Consider the zero-profit bid $\widehat{b}_i^T(\mathcal{M})$ under the menu $\mathcal{M}$. Then we have $\Pi_i^T(\widehat{b}_i^T(\mathcal{M});\mathcal{M}) = 0$ and there exists $f^* \in \mathcal{F}_i$ such that $\Pi_i(\widehat{b}_i^T(\mathcal{M}), f^*, \tilde{\sigma}(f^*);\mathcal{M}) = \mathbb{E}_{f^*}[U_i(T_{(\widehat{b}_i^T(\mathcal{M}),\tilde{\sigma}(f^*))}(q) - C_i(f^*))] = 0$.
Define the function $\bar{U}_i$ by $\bar{U}_i(x) := U_i(x - C_i(f^*))$ for any $x \in \mathbb{R}$. Note that the concavity/strict concavity properties of U_i carry over to $\bar{U}_i$. We then have $\mathbb{E}_{f^*}[\bar{U}_i(T_{(\widehat{b}_i^T(\mathcal{M}),\tilde{\sigma}(f^*))}(q))] = 0$. From Definition 1, if the menu of contracts $\mathcal{M}'$ provides more insurance than the menu $\mathcal{M}$, then we have $\mathbb{E}_{f^*}[\bar{U}_i(T'_{(\widehat{b}_i^T(\mathcal{M}),\tilde{\sigma}(f^*))}(q))] \geq 0$ and the inequality is strict if $\bar{U}_i$ is strictly concave (or equivalently if U_i is strictly concave). The inequality $\mathbb{E}_{f^*}[\bar{U}_i(T'_{(\widehat{b}_i^T(\mathcal{M}),\tilde{\sigma}(f^*))}(q))] \geq 0$ (resp. > 0) implies that $\Pi_i^T(\widehat{b}_i^T(\mathcal{M});\mathcal{M}') \geq 0$ (resp. > 0). Note that from the definition of zero-profit bids, we have $\Pi_i^T(\widehat{b}_i^T(\mathcal{M}');\mathcal{M}') = 0$ and that the function $b \mapsto \Pi_i^T(b;\mathcal{M}')$ is (strictly) increasing. Thus, we obtain that $\widehat{b}_i^T(\mathcal{M}') \leq \widehat{b}_i^T(\mathcal{M})$ and that the inequality is strict if firm i is strictly risk averse.

The proof is analogous for strategic firms and we give it below for completeness.

Case 2: strategic firms

Here, the additional premise is that the menu $\mathcal{M}$ is strongly strategy-proof. Consider a risk-averse firm i, i.e. such that U_i is concave. Consider the zero-profit bid $\widehat{b}_i^S(\mathcal{M})$ under the menu $\mathcal{M}$. Then we have $\Pi_i^S(\widehat{b}_i^S(\mathcal{M});\mathcal{M}) = 0$ and since $\mathcal{M}$ is strongly strategy-proof, there exists $f^* \in \mathcal{F}_i$ such that $\Pi_i(\widehat{b}_i^S(\mathcal{M}), f^*, \tilde{\sigma}(f^*);\mathcal{M}) = \mathbb{E}_{f^*}[U_i(T_{(\widehat{b}_i^S(\mathcal{M}),\tilde{\sigma}(f^*))}(q) - C_i(f^*))] = 0$.
Define the function $\bar{U}_i$ by $\bar{U}_i(x) := U_i(x - C_i(f^*))$ for any $x \in \mathbb{R}$. Note that the concavity/strict concavity properties of U_i carry over to $\bar{U}_i$. We then have $\mathbb{E}_{f^*}[\bar{U}_i(T_{(\widehat{b}_i^S(\mathcal{M}),\tilde{\sigma}(f^*))}(q))] = 0$. From Definition 1, if the menu of contracts $\mathcal{M}'$ provides more insurance than the menu $\mathcal{M}$, then we have $\mathbb{E}_{f^*}[\bar{U}_i(T'_{(\widehat{b}_i^S(\mathcal{M}),\tilde{\sigma}(f^*))}(q))] \geq 0$ and the inequality is strict if $\bar{U}_i$ is strictly concave (or equivalently if U_i is strictly concave). The inequality $\mathbb{E}_{f^*}[\bar{U}_i(T'_{(\widehat{b}_i^S(\mathcal{M}),\tilde{\sigma}(f^*))}(q))] \geq 0$ (resp. > 0) implies that $\Pi_i^T(\widehat{b}_i^S(\mathcal{M});\mathcal{M}') \geq 0$ (resp. > 0). Note also that $\Pi_i^S(b;\mathcal{M}') \geq \Pi_i^T(b;\mathcal{M}')$ for any bid b, reflecting that firms can further increase their expected payoff, e.g. by reporting $\sigma \neq \tilde{\sigma}(f^*)$. Note that from the definition of zero-profit bids, we have $\Pi_i^S(\widehat{b}_i^S(\mathcal{M}');\mathcal{M}') = 0$ and that the function $b \mapsto \Pi_i^S(b;\mathcal{M}')$ is (strictly) increasing. So we get that $\widehat{b}_i^S(\mathcal{M}') \leq \widehat{b}_i^S(\mathcal{M})$ and the inequality is strict if the firm i is strictly risk-averse.

B Proof of Proposition 6

"If" part – Suppose q is distributed according to $f \in \mathcal{F}_{all}(q_0)$. Let h_λ (resp. $h_{\lambda'}$) denote the distribution of the variable $\lambda q + \mu$ (resp. $\lambda' q + \mu + (\lambda - \lambda') \cdot q_0$). Note that
$\mathbb{E}_{x \sim h_\lambda}[x] = \lambda \cdot q_0 + \mu = \mathbb{E}_{x \sim h_{\lambda'}}[x]$, which satisfies the mean-preserving condition
$\mathbb{E}_{q \sim f}[\lambda' q + \mu + (\lambda - \lambda') \cdot q_0] = \lambda \cdot q_0 + \mu = \mathbb{E}_{q \sim f}[\lambda q + \mu]$.
We show below, assuming $0 < \lambda' < \lambda$, that the distribution $H_{\lambda'}$ exhibits second-order stochastic dominance over H_λ, i.e., that for all $x \in \mathbb{R}$

$$\int_{-\infty}^{x} H_{\lambda'}(u)du \leq \int_{-\infty}^{x} H_\lambda(u)du \tag{8}$$

where the inequality holds strictly over some part of the range. We then obtain as a corollary of second-order stochastic dominance that for any concave utility function U,
$\mathbb{E}_{x \sim h_{\lambda'}}[U(x)] \geq \mathbb{E}_{x \sim h_\lambda}[U(x)]$ or equivalently $\mathbb{E}_{q \sim f}[U(\lambda' q + \mu + (\lambda - \lambda') \cdot q_0)] \geq \mathbb{E}_{q \sim f}[U(\lambda q + \mu)]$ and these inequalities are strict if U is strictly concave (see e.g., Hirshleifer and Riley (1992) on stochastic dominance).[38]
On the contrary, note that by symmetry the inequalities are reversed when $\lambda' > \lambda$: we have $\mathbb{E}_f[U(\lambda' q + \mu + (\lambda - \lambda') \cdot q_0)] \leq \mathbb{E}_f[U(\lambda q + \mu)]$, and the inequality is strict if U is strictly concave. We then obtain that the $(\lambda', \mu + (\lambda - \lambda') \cdot q_0)$-linear contract (with $\lambda' > 0$) provides more insurance than the (λ, μ)-linear contract (with $\lambda > 0$) if and only if $\lambda' < \lambda$.

Proof of second-order stochastic dominance – Note that the definition of the PDFs h_λ and $h_{\lambda'}$ translates in terms of CDFs into the equalities: $H_\lambda(u) = F(\frac{u-\mu}{\lambda})$ and $H_{\lambda'}(u) = F(\frac{u-\mu-(\lambda-\lambda')\cdot q_0}{\lambda'})$ for all u. Note also that the inequalities $0 < \lambda' < \lambda$ imply that $\frac{u-\mu-(\lambda-\lambda')\cdot q_0}{\lambda'}$ is strictly less (resp. strictly greater) than $\frac{u-\mu}{\lambda}$ if $u < \lambda q_0 + \mu$ (resp. $u > \lambda q_0 + \mu$). Then we get $H_{\lambda'}(u) \leq H_\lambda(u)$ for any $u \leq \lambda q_0 + \mu$, and conversely $H_{\lambda'}(u) \geq H_\lambda(u)$ for any $u \geq \lambda q_0 + \mu$. Furthermore, these inequalities are strict if $h_\lambda(u) > 0$ or $h_{\lambda'}(u) > 0$.
If $x \leq \lambda q_0 + \mu$, we then obtain $\int_{-\infty}^{x} H_{\lambda'}(u)du \leq \int_{-\infty}^{x} H_\lambda(u)du$ and there exists $x \leq \lambda q_0 + \mu$ such that the inequality is strict (for this we can take $x = \lambda q + \mu$ with $q \leq q_0$ such that $f(q) > 0$ and such a production q exists since $\bar{q}_f = q_0$).
Now consider $x \geq \lambda q_0 + \mu$ and let $\bar{x}$ denote the upper bound of the support of the distribution h_λ, which is greater than the upper bound of the distribution $h_{\lambda'}$. We have
$\int_{-\infty}^{x} H_{\lambda'}(u)du - \int_{-\infty}^{x} H_\lambda(u)du = \int_{-\infty}^{\bar{x}}[H_{\lambda'}(u) - H_\lambda(u)]du - \int_{x}^{\bar{x}}[H_{\lambda'}(u) - H_\lambda(u)]du$.
Thanks to integration by parts, we have $\int_{-\infty}^{\bar{x}}[H_{\lambda'}(u) - H_\lambda(u)]du = \int_{-\infty}^{\bar{x}}[h_{\lambda'}(u) - h_\lambda(u)]udu = 0$

[38] There are mild variations in the definitions of "second-order stochastic dominance" in the literature. In line with Definition 1, Hirshleifer and Riley (1992) adopt a version which guarantees that the inequalities are strict when the utility function is strictly concave.

since the two distributions h_λ and $h_{\lambda'}$ have the same mean and a support included in $]-\infty, \bar{x}]$. Then we get

$$\int_{-\infty}^{x} H_{\lambda'}(u)du - \int_{-\infty}^{x} H_\lambda(u)du = -\int_{x}^{\bar{x}} [H_{\lambda'}(u) - H_\lambda(u)]du.$$

Since $H_{\lambda'}(u) \geq H_\lambda(u)$ for any $u \geq \lambda q_0 + \mu$, we then conclude that $\int_{-\infty}^{x} H_{\lambda'}(u)du \leq \int_{-\infty}^{x} H_\lambda(u)du$ for any $x \geq \lambda q_0 + \mu$. We have thus established the second-order stochastic dominance condition (8).

"Only if" part – Suppose T provides more insurance than the (λ, μ)-linear contract for any distribution $f \in \mathcal{F}_{all}(q_0)$. Let us show that T must be a (λ', μ')-linear contract with $(\lambda - \lambda') \cdot q_0 = (\mu' - \mu)$ and $0 < \lambda' < \lambda$.

Take any pair $q_L, q_H \in \mathbb{R}_+^*$ such that $q_L < q_0 < q_H$, and any $\epsilon > 0$ sufficiently small so that $q_L - \epsilon > 0$ and $q_L + \epsilon < q_H - \epsilon$. Let us introduce the distribution $f_{(q_L, q_H, \epsilon)}$, according to which q follows a uniform distribution over $[q_L - \epsilon, q_L + \epsilon]$ with probability $\frac{q_H - q_0}{q_H - q_L}$, or follows a uniform distribution over $[q_H - \epsilon, q_H + \epsilon]$ with probability $\frac{q_0 - q_L}{q_H - q_L}$. Formally, $f_{(q_L, q_H, \epsilon)}(q) = \frac{1}{2\epsilon} \cdot \frac{q_H - q_0}{q_H - q_L}$ if $q \in [q_L - \epsilon, q_L + \epsilon]$, $f_{(q_L, q_H, \epsilon)}(q) = \frac{1}{2\epsilon} \cdot \frac{q_0 - q_L}{q_H - q_L}$ if $q \in [q_H - \epsilon, q_H + \epsilon]$, and is equal to 0 elsewhere. We have $\mathbb{E}_{f_{(q_L, q_H, \epsilon)}}[q] = \frac{q_H - q_0}{q_H - q_L} \cdot q_L + \frac{q_0 - q_L}{q_H - q_L} \cdot q_H = q_0$, and therefore $f_{(q_L, q_H, \epsilon)} \in \mathcal{F}_{all}(q_0)$.

The assumption that T provides more insurance than the (λ, μ)-linear contract implies the mean-preserving condition $\mathbb{E}_{f_{(q_L, q_H, \epsilon)}}[T(q)] = \lambda q_0 + \mu$ for any triplet (q_L, q_H, ϵ), or equivalently

$$(q_H - q_0) \cdot \int_{q_L - \epsilon}^{q_L + \epsilon} \frac{T(q)}{2\epsilon} dq + (q_0 - q_L \cdot) \int_{q_H - \epsilon}^{q_H + \epsilon} \frac{T(q)}{2\epsilon} dq = (q_H - q_L) \cdot (\lambda q_0 + \mu).$$

By taking the derivative of this equality with respect to q_H, we get $\frac{1}{2\epsilon} \int_{q_L - \epsilon}^{q_L + \epsilon} T(q)dq + (q_0 - q_L)\frac{1}{2\epsilon}(T(q_H + \epsilon) - T(q_H - \epsilon)) = \lambda q_0 + \mu$. Then, taking the derivative of the latter equality with respect to q_L, we get that

$$\frac{T(q_L + \epsilon) - T(q_L - \epsilon)}{2\epsilon} = \frac{T(q_H + \epsilon) - T(q_H - \epsilon)}{2\epsilon} \tag{9}$$

for all $0 < \epsilon < \min\{q_L, \frac{q_H - q_L}{2}\}$.

The function T is continuously increasing and thus differentiable almost everywhere. Consider $q_L^* \in]0, q_0[$ such that T is differentiable at q_L^*. Applying (9) when $q_L = q_L^*$ and for any $q_H > q_0$ and taking the limit when $\epsilon \to 0$, we get that T is differentiable at q_H with $T'(q_H) = T'(q_L^*)$. Similarly, if we fix $q_H^* > q_0$ and apply (9) when $q_L < q_0$, then we get that T is differentiable at q_L with $T'(q_L) = T'(q_H^*) = T'(q_L^*)$. All in all, we get that T' is constant and thus that T is a linear function on $\mathbb{R}_+$.

Now, denoting $T(q) = \lambda' \cdot q + \mu'$, the mean-preserving condition $\mathbb{E}_f[T(q)] = \lambda' q_0 + \mu' = \lambda q_0 + \mu$, for any $f \in \mathcal{F}_{all}(q_0)$, implies that $(\lambda - \lambda') \cdot q_0 = (\mu' - \mu)$. As a by-product of the "If part", we showed above that the (λ', μ')-linear contract would fail to provide more insurance than the (λ, μ)-linear contract if $\lambda' > \lambda$. Finally, we must have $0 < \lambda' < \lambda$, which completes the proof.

C Proof of Proposition 8

"If" part – Suppose that q is distributed according to $f \in \mathcal{F}_{sym}(q_0)$. Let $h_{(\lambda,\mu)}$ (resp. h_T) denote the PDF of the variable $\lambda \cdot q + \mu$ (resp. $T(q)$, where T satisfies the three conditions from Proposition 8). Note that $\mathbb{E}_{x\sim h_{(\lambda,\mu)}}[x] = \lambda \cdot q_0 + \mu$.
Note that the support of $f \in \mathcal{F}_{sym}(q_0)$ with expected value q_0 is included in $[0, 2q_0]$. Using the change of variable $q = q_0(1+\epsilon)$ and the second condition (i.e.
$\frac{1}{2} \cdot T(q_0(1-\epsilon)) + \frac{1}{2} \cdot T(q_0(1+\epsilon)) = \lambda \cdot q_0 + \mu$, for any $\epsilon \in [0,1]$) for any $\epsilon \in [0,1]$, we obtain the following calculation:

$$\begin{aligned}
\mathbb{E}_{x\sim h_T}[x] = \mathbb{E}_f[T(q)] &= \int_0^{2q_0} T(q) f(q) dq \\
&= q_0 \int_{-1}^{1} T(q_0 \cdot (1+\epsilon)) f(q_0 \cdot (1+\epsilon)) d\epsilon \\
&= q_0 \int_0^1 \left[T(q_0 \cdot (1+\epsilon)) + T(q_0 \cdot (1-\epsilon))\right] f(q_0(1+\epsilon)) d\epsilon \\
&= q_0 \int_0^1 2(\lambda \cdot q_0 + \mu) \cdot f(q_0(1+\epsilon)) d\epsilon \\
&= (\lambda \cdot q_0 + \mu) \cdot \underbrace{\int_0^{2q_0} f(q) dq}_{=1} = \lambda \cdot q_0 + \mu.
\end{aligned}$$

We thus obtain that $\mathbb{E}_{x\sim h_{(\lambda,\mu)}}[x] = \mathbb{E}_{x\sim h_T}[x]$, i.e. that T satisfies the the mean-preserving property $\mathbb{E}_f[\lambda q + \mu] = \mathbb{E}_f[T(q)]$.

We show below that the distribution H_T exhibits second-order stochastic dominance over $H_{(\lambda,\mu)}$, i.e., that for all x

$$\int_{-\infty}^{x} H_T(u) du \leq \int_{-\infty}^{x} H_{(\lambda,\mu)}(u) du. \tag{10}$$

where the inequality holds strictly over some part of the range (namely, the neighborhood of q_0). We obtain then as a corollary of second-order stochastic dominance that for any concave utility function U, $\mathbb{E}_{x\sim h_T}[U(x)] \geq \mathbb{E}_{x\sim h_{(\lambda,\mu)}}[U(x)]$ or equivalently $\mathbb{E}_f[U(T(q))] \geq \mathbb{E}_f[U(\lambda q + \mu)]$ and these inequalities are strict if U is strictly concave. We obtain then that the contract T provides more insurance than the (λ, μ)-linear contract (with $\lambda > 0$).

Proof of second-order stochastic dominance – Note that the first condition $T(q) \geq \lambda \cdot q + \mu$ for any $q \leq q_0$ implies that $H_T(x) \leq H_{(\lambda,\mu)}(x)$ for any $x \leq \lambda q_0 + \mu$. Note that $H_T(\lambda q_0 + \mu) = H_{(\lambda,\mu)}(\lambda q_0 + \mu) = \frac{1}{2}$. From the third condition ($\int_{q_0(1-\epsilon)}^{q_0}[T(q) - (\lambda q + \mu)]dq > 0$ for any $\epsilon \in (0, 1]$) we obtain that $H_T(x) < H_{(\lambda,\mu)}(x)$ for x in the left neighborhood of $\lambda q_0 + \mu$. If $x \leq \lambda q_0 + \mu$, we obtain then $\int_{-\infty}^{x} H_T(u)du \leq \int_{-\infty}^{x} H_{(\lambda,\mu)}(u)du$ and the inequality is strict in the left neighborhood of $\lambda q_0 + \mu$.

Consider now $x \geq \lambda q_0 + \mu$ and let $\bar{x}$ denote the upper bound of the support of the distribution $h_{(\lambda,\mu)}$, which is greater than the upper bound of the distribution h_T. We have $\int_{-\infty}^{x} H_T(u)du - \int_{-\infty}^{x} H_{(\lambda,\mu)}(u)du = \int_{-\infty}^{\bar{x}}[H_T(u) - H_{(\lambda,\mu)}(u)]du - \int_{x}^{\bar{x}}[H_T(u) - H_{(\lambda,\mu)}(u)]du$.

By integration by parts, we obtain $\int_{-\infty}^{\bar{x}}[H_T(u) - H_{(\lambda,\mu)}(u)]du = \int_{-\infty}^{\bar{x}} u[h_T(u) - h_{(\lambda,\mu)}(u)]du = \mathbb{E}_{x\sim h_T}[x] - \mathbb{E}_{x\sim h_{(\lambda,\mu)}}[x] = \mathbb{E}_f[T(x)] - \mathbb{E}_f[\lambda \cdot x + \mu] = 0$ as shown above given that $f \in \mathcal{F}_{sym}$.

We obtain then that for any $x \geq \lambda \cdot q_0 + \mu$:

$$\int_{-\infty}^{x} H_T(u)du - \int_{-\infty}^{x} H_{(\lambda,\mu)}(u)du = -\int_{x}^{\bar{x}}[H_T(u) - H_{(\lambda,\mu)}(u)]du.$$

Conditions 1 and 2 combined also impose that $T(q) \leq \lambda \cdot q + \mu$ for any $q \geq q_0$, which implies that $H_T(x) \geq H_{(\lambda,\mu)}(x)$ for any $x \geq \lambda q_0 + \mu$. So we conclude that $\int_{-\infty}^{x} H_T(u)du \leq \int_{-\infty}^{x} H_{(\lambda,\mu)}(u)du$ for any $x \geq \lambda q_0 + \mu$. We have thus established the second-order stochastic dominance condition (10).

"Only if" part

Suppose that T provides more insurance than the (λ, μ)-linear contract for any distribution $f \in \mathcal{F}_{sym}(q_0)$. For any $\epsilon \in]0, 1]$, let $f_{(q_0,\epsilon)}$ be the uniform distribution on the interval $[q_0(1 - \epsilon), q_0(1 + \epsilon)]$. We have that $f_{(q_0,\epsilon)} \in \mathcal{F}_{sym}(q_0)$. Formally, $f_{(q_0,\epsilon)}(q) = \frac{1}{2\cdot\epsilon\cdot q_0}$ if $q \in [q_0(1 - \epsilon), q_0(1 + \epsilon)]$ and $f_{(q_0,\epsilon)}(q) = 0$ elsewhere.

The assumption that T provides more insurance than the (λ, μ)-linear contract implies, for all $\epsilon \in]0, 1]$, the mean-preserving condition $\mathbb{E}_{f_{(q_0,\epsilon)}}[q] = \lambda \cdot q_0 + \mu$, or:

$$\int_{q_0(1-\epsilon)}^{q_0(1+\epsilon)} \frac{T(q)}{2 \cdot \epsilon \cdot q_0} \cdot dq = \lambda q_0 + \mu.$$

By multiplying each side by ϵ and taking the derivative of this equality with respect to ϵ, we get

$$\frac{1}{2} \cdot T(q_0(1 - \epsilon)) + \frac{1}{2} \cdot T(q_0(1 + \epsilon)) = \lambda q_0 + \mu. \tag{11}$$

By continuity, the equality holds at the bounds and thus for any $\epsilon \in [0, 1]$. Thus we have shown condition 2 in Proposition 8.

In order to show that $T(q) \leq \lambda q + \mu$ for any $q \in [q_0, 2q_0]$ or equivalently that $T(q_0(1 + \epsilon)) \leq \lambda q_0(1 + \epsilon) + \mu$ for any $\epsilon \in [0, 1]$, let us proceed by contradiction. Suppose on the

contrary that $T(q_0(1+\epsilon)) > \lambda q_0(1+\epsilon)+\mu$ for some $\epsilon \in [0,1]$ and let then $\underline{\delta} := \inf\{\epsilon \in [0,1] \,|\, T(q_0(1+\epsilon)) > \lambda q_0(1+\epsilon)+\mu\}$. Since T is continuous, we have then $\underline{\delta} < 1$ and we can also define $\overline{\delta} \in (\underline{\delta}, 1]$ such that $T(q_0(1+\epsilon)) > \lambda q_0(1+\epsilon)+\mu$ for any $\epsilon \in]\underline{\delta}, \overline{\delta}[$. Since T is continuous, we also have $T(q_0(1+\underline{\delta})) = \lambda q_0(1+\underline{\delta})+\mu$.
Consider then $f_{(q_0,\overline{\delta})}$ the uniform distribution on $[q_0(1-\overline{\delta}), q_0(1+\overline{\delta})]$. Consider a continuous function U such that $U(x)=x$ for $x \leq \lambda q_0(1+\underline{\delta})+\mu$ and $U'(q) \in]0,1[$ being strictly decreasing for $q > \lambda q_0(1+\underline{\delta})+\mu$.[39] Note that U is then increasing and concave.
Given that T is an increasing function and that $T(q_0(1+\underline{\delta})) = \lambda q_0(1+\underline{\delta})+\mu$ (which also implies $T(q_0(1-\underline{\delta})) = \lambda q_0(1-\underline{\delta})+\mu$ given (11)), we have that $T(q) \in [\lambda q_0(1-\underline{\delta})+\mu, \lambda q_0(1+\underline{\delta})+\mu]$ for any $q \in [q_0(1-\underline{\delta}), q_0(1+\underline{\delta})]$. Therefore using that $U(x)=x$ for $x \in [\lambda q_0(1-\underline{\delta})+\mu, \lambda q_0(1+\underline{\delta})+\mu]$, the symmetry of $f_{(q_0,\overline{\delta})}$ around q_0, and making the change of variable $\epsilon = \frac{q}{q_0}-1$ in (11) we obtain:

$$\begin{aligned}
\int_{q_0(1-\underline{\delta})}^{q_0(1+\underline{\delta})} U(T(q)) f_{(q_0,\overline{\delta})}(q)dq &= \int_{q_0(1-\underline{\delta})}^{q_0(1+\underline{\delta})} T(q) F_{(q_0,\overline{\delta})}(q)dq \\
&= q_0 \cdot \int_0^{\underline{\delta}} [T(q_0(1+\epsilon)) + T(q_0(1-\epsilon))] f_{(q_0,\overline{\delta})}(q_0(1+\epsilon))d\epsilon \\
&= 2[\lambda q_0+\mu] \cdot [F_{(q_0,\overline{\delta})}(q_0(1+\underline{\delta})) - \frac{1}{2}] \\
&= [\lambda q_0+\mu] \cdot [F_{(q_0,\overline{\delta})}(q_0(1+\underline{\delta})) - F_{(q_0,\overline{\delta})}(q_0(1-\underline{\delta}))] \\
&= \int_{q_0(1-\underline{\delta})}^{q_0(1+\underline{\delta})} [\lambda q_0+\mu] dF_{(q_0,\overline{\delta})}(q) = \int_{q_0(1-\underline{\delta})}^{q_0(1+\underline{\delta})} (\lambda q+\mu) \cdot dF_{(q_0,\overline{\delta})}(q) \\
&= \int_{q_0(1-\underline{\delta})}^{q_0(1+\underline{\delta})} U(\lambda q+\mu) dF_{(q_0,\overline{\delta})}(q).
\end{aligned}$$

Note that the first and the last equalities use the assumption that U is linear on $[0, q_0 \cdot (1+\underline{\delta})]$. We obtain thus that the difference $\mathbb{E}_{f_{(q_0,\overline{\delta})}}[U(\lambda q+\mu)] - \mathbb{E}_{f_{(q_0,\overline{\delta})}}[U(T(q))]$ resumes to

$$\int_{q_0(1-\overline{\delta})}^{q_0(1-\underline{\delta})} [U(\lambda q+\mu) - U(T(q))] \frac{dq}{2\overline{\delta} q_0}$$

$+ \int_{q_0(1+\underline{\delta})}^{q_0(1+\overline{\delta})} [U(\lambda q+\mu) - U(T(q))] \frac{dq}{2\overline{\delta} q_0}$.Thanks to the change of variable $\epsilon = 1-\frac{q}{q_0}$ and $\epsilon = \frac{q}{q_0}-1$ in the first and second integrals, respectively, we obtain:

[39] How to build a function U satisfying such properties (which will guarantee then its existence) is left to the reader.

$$\begin{aligned}\mathbb{E}_{f_{(q_0,\overline{\delta})}}[U(\lambda q+\mu)]-\mathbb{E}_{f_{(q_0,\overline{\delta})}}[U(T(q))] =& \frac{1}{2\overline{\delta}}\int_{\underline{\delta}}^{\overline{\delta}}[U(\lambda q_0(1-\epsilon)+\mu)-U(T(q_0(1-\epsilon)))]d\epsilon \\ &+\frac{1}{2\overline{\delta}}\int_{\underline{\delta}}^{\overline{\delta}}[U(\lambda q_0(1+\epsilon)+\mu)-U(T(q_0(1+\epsilon)))]d\epsilon.\end{aligned} \tag{12}$$

Let us show below that in the first (resp. second) integral the function U is applied to values where it is linear (resp. strictly concave). Since the function T is increasing, for any $\epsilon \in [0,1]$ we have $T(q_0(1-\epsilon)) \leq T(q_0) = \lambda \cdot q_0 + \mu$ where the last equality comes from (11)). In the first integral, the function U is thus applied only for values below $q_0(1+\underline{\delta})+\mu$ where the function U is defined such that $U(x)=x$. We have thus that $\forall \epsilon \in [\underline{\delta},\overline{\delta}]$:

$$U(\lambda q_0(1-\epsilon)+\mu)-U(T(q_0(1-\epsilon))) = \lambda q_0(1-\epsilon)+\mu-T(q_0(1-\epsilon)). \tag{13}$$

Since the function T is increasing and $T(q_0(1+\underline{\delta})) = \lambda q_0(1+\underline{\delta})+\mu$ (from the way we have defined $\underline{\delta}$), then for $\epsilon \in [\underline{\delta},\overline{\delta}]$, we have that $T(q_0(1+\epsilon)) \geq T(q_0(1+\underline{\delta})) = \lambda q_0(1+\underline{\delta})+\mu$. Besides, we note that $\lambda q_0(1+\epsilon)+\mu \geq \lambda q_0(1+\underline{\delta})+\mu$. In the second integral, the function U is thus applied only for values above $\lambda q_0(1+\underline{\delta})+\mu$ where the function U is strictly concave and with $U'(x)<1$. We have thus that $\forall \epsilon \in (\underline{\delta},\overline{\delta}]$ where $T(q_0(1+\epsilon)) \geq \lambda q_0(1+\epsilon)+\mu$:

$$\begin{aligned}U(T(q_0(1+\epsilon)))-U(\lambda q_0(1+\epsilon)+\mu) &\leq [T(q_0(1+\epsilon))-\lambda q_0(1+\epsilon)+\mu]\cdot U'(\lambda q_0(1+\epsilon)+\mu) \\ &< \lambda q_0(1+\epsilon)+\mu-T(q_0(1+\epsilon)).\end{aligned} \tag{14}$$

We obtain thus that

$$U(\lambda q_0(1+\epsilon)+\mu)-U(T(q_0(1+\epsilon))) > \lambda q_0(1+\epsilon)+\mu-T(q_0(1+\epsilon)). \tag{15}$$

Finally, plugging (13) and (15) into (12) and using (11), we obtain:

$$\mathbb{E}_{f_{(q_0,\overline{\delta})}}[U(\lambda q+\mu)]-\mathbb{E}_{f_{(q_0,\overline{\delta})}}[U(T(q))] > \frac{1}{2\overline{\delta}}\int_{\underline{\delta}}^{\overline{\delta}}\underbrace{[2(\lambda q_0+\mu)-T(q_0(1-\epsilon))-T(q_0(1+\epsilon))]}_{=0}d\epsilon = 0.$$

We have thus shown that $\mathbb{E}_{f_{(q_0,\overline{\delta})}}[U(\lambda q+\mu)] > \mathbb{E}_{f_{(q_0,\overline{\delta})}}[U(T(q))]$, which stands in contradiction with the production-insuring condition (3). On the whole we have shown that $T(q_0(1+\epsilon)) \leq \lambda q_0(1+\epsilon)+\mu$ for any $\epsilon \in [0,1]$. From the symmetry condition (11), we obtain $T(q_0(1-\epsilon)) \geq \lambda q_0(1-\epsilon)+\mu$ for any $\epsilon \in [0,1]$.
The remaining part of Proposition 8 to be shown is that $T(q)-[\lambda q+\mu]$ can not be equal

(uniformly) to zero in the neighborhood of q_0 or equivalently (given that we have shown that $T(q_0(1-\epsilon)) - [\lambda q_0(1-\epsilon)+\mu] \geq 0$ for any $\epsilon \in [0,1]$ and that T is continuous) that for all $\epsilon \in]0,1]$ we verify $\int_{q_0(1-\epsilon)}^{q_0}[T(q)-(\lambda q+\mu)]dq > 0$ for any $\epsilon \in (0,1]$.
Suppose that $T(q) - [\lambda q + \mu]$ for any $q \in [q_0(1-\epsilon), q_0(1+\epsilon)]$ (with $\epsilon > 0$) and let us establish a contradiction. Consider a strictly concave payoff function U and the uniform distribution $f_{(q_0,\epsilon)}$. Since $T(q)$ is uniformly equal to $\lambda q + \mu$ on the support of $f_{(q_0,\epsilon)}$, then we obtain that $\mathbb{E}_{f_{(q_0,\epsilon)}}[U(\lambda q)+\mu] = \mathbb{E}_{f_{(q_0,\epsilon)}}[U(T(q))]$ which stands in contradiction with the production-insuring inequality (3) which must be strict if U is strictly concave.

D Proof of Proposition 9

Let us establish Proposition 9 for $q_0 > \bar{q}$. This is the case we use for our illustration of the proof in Figure 2. The proof for $q_0 < \bar{q}$ is analog by symmetry and briefly discussed at the end.
Summary of the arguments: The central point is to decompose $f \in \mathcal{F}_{sym}(\bar{q})$ as $f = \alpha \cdot g + (1-\alpha) \cdot h$ where $\alpha \in [0,1[$ and g and h are two PDFs such that $g \in \mathcal{F}_{sym}(q_0)$ and the support of h is included in $[0, q_0]$. The assumption that f is single-peaked plays a key role to guarantee that h is a PDF. This gives the decompositions:

$$\mathbb{E}_f[q] = \bar{q} = \alpha \cdot q_0 + (1-\alpha) \cdot \mathbb{E}_h[q] \quad \text{and} \quad \mathbb{E}_f[T(q)] = \alpha \cdot \mathbb{E}_g[T(q)] + (1-\alpha) \cdot \mathbb{E}_h[T(q)] \tag{16}$$

Since $g \in \mathcal{F}_{sym}(q_0)$, the assumption that T provides more insurance on $\mathcal{F}_{sym}(q_0)$ implies that $\mathbb{E}_g[T(q)] = \lambda \cdot q_0 + \mu$. Since $T(q) \geq \lambda \cdot q + \mu$ for any $q \in [0, q_0]$ (Proposition 8) and since the support of h is included in $[0, q_0]$, we obtain that $\mathbb{E}_h[T(q)] \geq \lambda \cdot \mathbb{E}_h[q] + \mu$. From (16), we obtain then that $\mathbb{E}_f[T(q)] \geq \lambda \cdot [\alpha \cdot q_0 + (1-\alpha) \cdot \mathbb{E}_h[q]] + \mu = \lambda \cdot \mathbb{E}_f[q] + \mu$.
Furthermore, the additional assumption $f'(q_0) < 0$ implies that the PDF h has a positive mass in the left neighborhood of q_0 and thanks to Condition 3 in Proposition 8, we obtain then that $\mathbb{E}_h[T(q)] > \lambda \cdot \mathbb{E}_h[q] + \mu$ and thus that $\mathbb{E}_f[T(q)] > \lambda \cdot \mathbb{E}_f[q] + \mu$.

Detailed arguments Formally, α, g and h are defined in the following way. We let $\alpha := 2 \cdot (1 - F(q_0)) < 2 \cdot (1 - F(\bar{q})) = 1$. If $f'(q_0) < 0$ (as it is the case in Figure 2), then q_0 belongs to the interior of the support of f and $F(q_0) < 1$ or equivalently $\alpha > 0$.
If $\alpha = 0$ or equivalently $F(q_0) = 1$, we let $h := f$ and the support of h is included in $[0, q_0]$.
If $\alpha > 0$, let $G : \mathbb{R}_+ \mapsto [0,1]$ denote the function defined by:

$$\text{for } q \geq q_0, \quad G(q) = \frac{1 + F(q) - 2F(q_0)}{2 - 2F(q_0)}$$
$$\text{for } q < q_0, \quad G(q) = 1 - \frac{1 + F(2q_0 - q) - 2F(q_0)}{2 - 2F(q_0)}.$$

As a CDF, the function F is non-decreasing, we obtain then that G is non-decreasing on $[0, q_0[$ and on $[q_0, +\infty[$. In addition, G is continuous at q_0 with $G(q_0) = \frac{1}{2}$, and $G(q) = 0$ if $q \leq 2(q_0 - \bar{q})$, and $G(q) = 1$ if $q \geq 2\bar{q}$. We obtain then that G is non-decreasing on $\mathbb{R}_+$. Note also that G is differentiable and the derivative, denoted by g, satisfies $g(q) = \frac{f(q)}{\alpha}$ if $q \geq q_0$ and $g(q) = \frac{f(2q_0 - q)}{\alpha}$ if $q \leq q_0$. We have then $g(q_0 + x) = g(q_0 - x)$ for any $x \in [0, q_0]$. Furthermore, the single-peakness property of f implies that g is non-increasing on $[q_0, 2q_0]$ and by symmetry g is non-decreasing on $[0, q_0]$. If $q > 2q_0$, we have $q > 2\bar{q}$ and then $g(q) = \frac{f(q)}{\alpha} = 0$. On the whole, we have shown that $g \in \mathcal{F}_{sym}(q_0)$.
Let us define the function $h : \mathbb{R}_+ \mapsto \mathbb{R}$ by $h = \frac{f - \alpha g}{1 - \alpha}$. Since $f(q) = \alpha \cdot g(q)$ for any $q \geq q_0$, we have $h(q) = 0$ if $q \geq q_0$. Let $H(q) := \int_0^q h(x)dx$. Note that $H(q_0) = H(2q_0) = \frac{1}{1-\alpha} \cdot [F(2q_0) - \alpha G(2q_0)] = 1$. To conclude that h is a CDF on $[0, q_0]$, we need to show in addition that $h(q) \geq 0$ for any $q \in [0, q_0]$. If $q \in [0, 2q_0 - 2\bar{q}[$, then $g(q) = g(2q_0 - q)$ where $2q_0 - q > 2\bar{q}$ which implies that $g(2q_0 - q) = 0$. We obtain then that $h(q) \geq 0$ if $q \in [0, 2q_0 - 2\bar{q}[$. Consider $q \in [2\bar{q} - q_0, q_0]$. Since f (resp. g) is single-peaked and symmetric around its mode $\bar{q}$ (resp. q_0), we have $f(q) \geq f(q_0)$ (resp. $g(q) \leq g(q_0)$) for any $q \in [2\bar{q} - q_0, q_0]$. We obtain then that $h(q) = \frac{f(q) - \alpha g(q)}{1 - \alpha} \geq \frac{f(q_0) - \alpha g(q_0)}{1 - \alpha} = 0$. If $2q_0 - 2\bar{q} > 2\bar{q} - q_0$, then we obtain that $h(q) \geq 0$ for any $q \in [0, q_0]$.
Consider now the case where $3q_0 > 4\bar{q}$ and consider $q \in [2q_0 - 2\bar{q}, 2\bar{q} - q_0]$. Since f (resp. g) is single-peaked and symmetric around its mode $\bar{q} > 2\bar{q} - q_0$ (resp. $q_0 > 2\bar{q} - q_0$), we have $f(q) \geq f(2q_0 - 2\bar{q})$ (resp. $g(q) \leq g(2\bar{q} - q_0)$) for any $q \in [2\bar{q} - q_0, q_0]$. In order to show that $h(q) \geq 0$ on $[2q_0 - 2\bar{q}, 2\bar{q} - q_0]$, it is then sufficient to show that $f(2q_0 - 2\bar{q}) \geq g(2\bar{q} - q_0)$. Note that the symmetry properties implies that $f(2q_0 - 2\bar{q}) = f(3\bar{q} - 2q_0)$ (f is symmetric around $\bar{q}$) and $g(2\bar{q} - q_0) = g(3q_0 - 2\bar{q}) = f(3q_0 - 2\bar{q})$ (g is symmetric around q_0 and $g(q) = f(q)$ for $q \geq q_0$). Note that $f(3\bar{q} - 2q_0) \geq f(3q_0 - 2\bar{q})$ because $\bar{q} \geq 3\bar{q} - 2q_0 > 3q_0 - 2\bar{q}$. We have thus shown that $h(q) \geq 0$ on $[2q_0 - 2\bar{q}, 2\bar{q} - q_0]$.

Let us now use the decomposition $f = \alpha \cdot g + (1 - \alpha)h$ to show that $\mathbb{E}_f[T(q)] \geq \lambda \cdot \bar{q} + \mu$ with a strict inequality if $f'(q_0) < 0$. Since the contract T provides more insurance than the (λ, μ)-linear contract on $\mathcal{F}_{sym}(q_0)$, we obtain 1) that $\mathbb{E}_g[T(q)] = \lambda \cdot q_0 + \mu$ (because $g \in \mathcal{F}_{sym}(q_0)$) and 2) that

$$\mathbb{E}_h[T(q)] = \int_0^{q_0} T(q)h(q)dq \geq \lambda \cdot \mathbb{E}_h[q] + \mu \tag{17}$$

(because Condition 1 in Proposition 8 guarantees that $T(q) \geq \lambda \cdot \bar{q} + \mu$ if $q \in [0, \bar{q}]$).
Given the decomposition $f = \alpha \cdot g + (1-\alpha)$, we get
$\mathbb{E}_f[T(q)] = \alpha \cdot \mathbb{E}_g[T(q)] + (1-\alpha) \cdot \mathbb{E}_h[T(q)] \geq \lambda \cdot [\alpha \mathbb{E}_g[q] + (1-\alpha)\mathbb{E}_h[q]] + \mu = \lambda \cdot \bar{q} + \mu$. If $f'(q_0) < 0$, then we have indeed $h(q) > 0$ for any $q \in [2\bar{q} - q_0, q_0[$. From condition 3 in Proposition 8, we have $\int_0^{\bar{q}} T(q)h(q)dq > \lambda \cdot \int_0^{\bar{q}} h(q)dq + \mu = \lambda \cdot \mathbb{E}_h[q] + \mu$ thanks to the positive mass of the distribution h in the left-neighborhood of $\bar{q}$. Finally, we obtain that $\mathbb{E}_f[T(q)] > \lambda \cdot \bar{q} + \mu$.

The proof for the case $q_0 < \bar{q}$ is analogous. There is however a small twist. In the decomposition $f = \alpha g + (1-\alpha)h$, as above, we have $\alpha \in [0, 1[$, $g \in \mathcal{F}_{sym}(q_0)$ and the support of the distribution h is included in $[q_0, 2\bar{q}]$ (instead of $[0, q_0]$ in the case $q_0 > \bar{q}$). From Proposition 8, we have that $T(q) \leq \lambda \cdot q + \mu$ if $q \in [q_0, 2q_0]$. Since we also assume that $T(q) \leq \lambda \cdot q + \mu$ for $q \geq 2 \cdot q_0$, the same logic of the proof applies.

E Proof of Theorem 10

Let us first explain why some technical assumptions need to be introduced to guarantee that firms' optimal strategies are well-defined. Consider the production-insuring menu of contracts $T_{(b,q_0)}(q) = \lambda^*(b, q_0) \cdot q + \mu(b) + (\lambda(b) - \lambda^*(b, q_0)) \cdot q_0$ with $0 < \lambda^*(b, q_0) < \lambda(b)$. If the function $q_0 \mapsto \lambda^*(b, q_0)$ does not depend on q_0, then let $\lambda^*(b, q_0) \equiv \lambda^*(b)$. We obtain then that the expected payoff of a risk neutral contractor as a function of the reference production q_0 is equal to $(\lambda(b) - \lambda^*(b)) \cdot q_0$ up to a constant, and is thus increasing in q_0 for any given pair (b, f). In such a case, firms' optimal strategies are not well-defined (which illustrates the need for extra technical assumptions as argued in Footnote 8). In order to guarantee that the sets $Q_i^*(b) := \operatorname{Arg\,max}_{(f,q_0) \in \mathcal{F}_i \times \{q_0 \in \mathbb{R}_+ | \exists f \in \mathcal{F} \text{ such that } q_0 = \bar{q}_f\}} \Pi_i(b, f, q_0)$ are well-defined and compact, we assume in the formal version of Theorem 10 that the sets $\mathcal{F}_i$ are finite and we limit ourselves to a portfolio $\mathcal{F}$ of projects whose expectations belongs to an interval $[q^{min}, q^{max}]$ with $q^{max} > q^{min} > 0$.

Theorem 13 (Formal version of Theorem 10)**.** *Consider a menu* $\mathcal{M} := \{T_{(b,q_0)}\}_{(b,q_0) \in \mathbb{R} \times [q^{min}, q^{max}]}$ *that is production-insuring on* $\mathcal{F} = \{f \in \mathcal{F}_{all} | \bar{q}_f \in [q^{min}, q^{max}]\}$ *(resp.* $\mathcal{F} = \{f \in \mathcal{F}_{sym} | \bar{q}_f \in [q^{min}, q^{max}]\}$*) against the menu of linear contracts* $\{\lambda(b) \cdot q + \mu(b)\}_{b \in \mathbb{R}}$.
Suppose that the set $\mathcal{F}_i \subseteq \mathcal{F}$ *is finite for each* i *and that* $\bar{q}_f < q^{max}$ *and that* f *is continuously differentiable for any* $f \in \mathcal{F}_i$.
For any firm, any bid and any cost function, the procurement is manipulable if the firm is risk neutral or if its risk aversion is small enough.

In equilibrium, if firms are risk neutral (or if firms' risk aversion is small enough), the contractor's expected production differs from its declaration. If we further assume that $T_{(b,q_0)}(q) \leq \lambda(b) \cdot q + \mu(b)$ for any pair q, q_0 with $q > 2q_0$,[40] *then strategic firms overstate their production and the buyer suffers from deception if the contractor is a strategic firm and if its bid b satisfies $\lambda(b) \leq v$.*

Proof of Theorem 13

Case 1: $\mathcal{F} = \{f \in \mathcal{F}_{all} | \bar{q}_f \in [q^{min}, q^{max}]\}$ and $\mathcal{M} := \{T_{(b,q_0)}\}_{(b,q_0) \in \mathbb{R} \times [q^{min}, q^{max}]}$ is production-insuring on $\mathcal{F}$ against the menu of linear contracts $\{\lambda(b) \cdot q + \mu(b)\}_{b \in \mathbb{R}}$.
Given the production-insuring property, we can write
$T_{(b,q_0)}(q) = \lambda'(b, q_0) \cdot q + \mu(b) + (\lambda(b) - \lambda'(b, q_0)) \cdot q_0$ where $\lambda'(b, q_0) < \lambda(b)$.
For a given firm i, we have thus

$$\begin{aligned}\Pi_i(b, f, q_0) - \Pi_i(b, f, \bar{q}) =& \mathbb{E}_f[U_i(\lambda'(b, q_0) \cdot q + \mu(b) + (\lambda(b) - \lambda'(b, q_0)) \cdot q_0) - C_i(f))] \\ &- \mathbb{E}_f[U_i(\lambda'(b, \bar{q}) \cdot q + \mu(b) + (\lambda(b) - \lambda'(b, \bar{q})) \cdot \bar{q}) - C_i(f))].\end{aligned}$$

Which, under risk neutrality, reduces to:

$$\Pi_i(b, f, q_0) - \Pi_i(b, f, \bar{q}) = (\lambda'(b, q_0) - \lambda(b)) \cdot (\bar{q} - q_0).$$

We thus have $\Pi_i(b, f, q_0) > \Pi_i(b, f, \bar{q})$ for $\bar{q} < q_0$, and thus $\Pi_i^S(b) > \Pi_i^T(b)$ (where strategic firms declare some $q_0 > \bar{q}$):[41] the procurement is manipulable at b for firm i.
Since $\mathcal{F}_i$ is finite and $[\bar{q}_f, q^{max}]$ is compact and $q_0 \mapsto \Pi_i(b, f, q_0)$ is continuous, we obtain that the set $Q_i^*(b) = \operatorname{Arg\,max}_{(f,q_0)|f \in \mathcal{F}_i, q_0 \in [\bar{q}_f, q^{max}]} \Pi_i(b, f, q_0)$ is a well-defined compact set.
Furthermore, for any i and any b, any pair (f, q^*) in $Q_i^*(b)$ satisfies $q^* > \bar{q}_f$. In equilibrium, the decision (b, f, q^*) of any strategic bidder should thus satisfy the property $q^* > \bar{q}_f^*$. If the contractor is strategic and if $\lambda(b) < v$ which further implies that $\lambda(b, q^*) < v$, then the buyer suffers from deception: the buyer's expected payoff under the contract $T_{(b,q^*)}$ when the contractor chooses project f is equal to
$v \cdot \bar{q}_f - \mathbb{E}_f[\lambda(b, q^*)q + \mu(b) + (\lambda(b) - \lambda(b, q^*)) \cdot q^*] = (v - \lambda(b, q^*)) \cdot \bar{q}_f - [\lambda(b)q^* + \mu(b)]$ which is smaller than
$\min_{f' \in \mathcal{F} | \bar{q}_{f'} = q^*} \{v \cdot \bar{q}_{f'} - \mathbb{E}_{f'}[\lambda(b, q^*)q + \mu(b) + (\lambda(b) - \lambda(b, q^*)) \cdot q^*]\} = (v - \lambda(b, q^*)) \cdot q^* - [\lambda(b) \cdot q^* + \mu(b)]$, since $v - \lambda(b, q^*) > 0$ and $q^* > \bar{q}$. This means that the buyer suffers from deception.

[40] This extra assumption is indeed always satisfied in the case where $\mathcal{F} = \{f \in \mathcal{F}_{all} | \bar{q}_f \in [q^{min}, q^{max}]\}$. On the contrary, it is needed when $\mathcal{F} = \{f \in \mathcal{F}_{sym} | \bar{q}_f \in [q^{min}, q^{max}]\}$ because the characterization of production-insuring menus does not impose any constraint on $T_{(b,q_0)}(q)$ for $q > 2q_0$.
[41] We use here that $\bar{q}_f < q^{max}$ for any $f \in \mathcal{F}_i$.

Thanks to Berge's Maximum Theorem (Berge, 1963), the overestimation result under risk neutrality extends to risk aversion, provided that risk aversion is small enough. Formally, once the underlying utility function space is equipped with a proper topology (consider, e.g. that the set of utility functions U_γ is parameterized by a parameter $\gamma \in \mathbb{R}^K$ such that $U_{(0,\cdots,0)}(x)$ corresponds to the risk-neutral case), we note that for any pair (b, f), the function $(q_0, U) \mapsto \Pi_i(b, f, q_0)$ is continuous, in particular given the linearity structure with respect to the utility function U given the expected utility criterion. Berge's Maximum Theorem implies that the correspondence mapping the utility function to the set of maximizers $Q_i^*(b)$ is upper-hemicontinuous (with non-empty and compact values): for any given sequence of utility functions that converges to the risk-neutral utility function (i.e. to $U(x) = x$), then any convergent sequence in the corresponding sets of maximizers has a limit belonging to the risk-neutral set of maximizers.

Note that the overestimation result holds for any pair (b, f), and thus in particular for the equilibrium pair. Deception also holds for limited risk aversion, insofar as overestimation implies deception (Corollary 7).

Case 2: $\mathcal{F} = \{f \in \mathcal{F}_{sym} | \bar{q}_f \in [q^{min}, q^{max}]\}$ and $\mathcal{M} := \{T_{(b,q_0)}\}_{(b,q_0)\in\mathbb{R}\times[q^{min},q^{max}]}$ is production-insuring on $\mathcal{F}$ against the menu of linear contracts $\{\lambda(b) \cdot q + \mu(b)\}_{b\in\mathbb{R}}$. Under risk neutrality and given the mean-preserving condition under truthful reporting, we have for any given firm i

$$\begin{aligned}\Pi_i(b, f, q_0) - \Pi_i(b, f, \bar{q}) &= \mathbb{E}_f[T_{(b,q_0)}(q) - C_i(f)] - \mathbb{E}_f[T_{(b,\bar{q})}(q) - C_i(f)] \\ &= \mathbb{E}_f[T_{(b,q_0)}(q)] - [\lambda(b) \cdot \bar{q}_f + \mu(b)].\end{aligned}$$

We have from Proposition 9 that $\mathbb{E}_f[T_{(b,q_0)}(q)] \geq \lambda(b) \cdot \bar{q}_f + \mu(b)$ if $q_0 \geq \bar{q}_f$ and that $\mathbb{E}_f[T_{(b,q_0)}(q)] \leq \lambda(b) \cdot \bar{q}_f + \mu(b)$ if $q_0 \leq \bar{q}_f$. Furthermore, for any f, there exists $q_0 > \bar{q}_f$ such that $f'(q_0) < 0$ (formally, $f'(q_0) \leq 0$ for any $q_0 \geq \bar{q}_f$, since f is single-peaked and f' cannot be uniformly equal to zero). We thus obtain that $\Pi_i(b, f, q_0) > \Pi_i(b, f, \bar{q})$ for some $q_0 > \bar{q}$, and thus $\Pi_i^S(b) > \Pi_i^T(b)$ (with strategic firms declaring some $q_0 > \bar{q}$): the procurement is manipulable at b for firm i.

As in case 1, we also obtain that for any bid b and any firm i, under risk neutrality, the set of maximizers $Q_i^*(b) = \operatorname{Arg\,max}_{(f,q_0)|f\in\mathcal{F}_i, q_0\in[\bar{q}_f,q^{max}]} \Pi_i(b, f, q_0)$ is a well-defined compact set such that any pair $(f, q^*) \in Q_i^*(b)$ satisfies $q^* > \bar{q}_f$. Consider then that the difference between the buyer's expected payoff $v \cdot \bar{q}_f - \mathbb{E}_f[T_{(b,q^*)}(q)]$ (given the pair (f, q^*) chosen by the contractor) and the expected payoff from the buyer's perspective, assuming the contractor is truthful in reporting q^*, is equal to $v \cdot q^* - [\lambda(b)q^* + \mu(b)] = v \cdot \bar{q}_f + (v - \lambda(b)) \cdot (q^* - \bar{q}_f) - [\lambda(b)\bar{q}_f + \mu(b)]$. Here the inequality $\Pi_i^S(b) > \Pi_i^T(b)$ implies that $\mathbb{E}_f[T_{(b,q^*)}(q) > \mathbb{E}_f[T_{(b,\bar{q}_f)}(q) = [\lambda(b)\bar{q}_f + \mu(b)]$.

Since we also have $(v - \lambda(b)) \cdot (q^* - \bar{q}_f) \geq 0$ (given our assumption that $\lambda(b) \leq v$), we obtain that the buyer suffers deception when the contractor is a strategic firm.
As in Case 1, Berge's Maximum Theorem guarantees that for any firm i and any possible bid b, the firm, if strategic, strictly overstates its production when risk aversion is limited.
Furthermore, the difference between the buyer's expected payoff and the payoff she expects with the mistaken belief that the contractor is truthful is continuous in the utility function, guaranteeing that deception also holds with limited risk aversion. **Q.E.D.**

Supplementary Appendix

(for online publication only)

The Supplementary Appendix is composed of three appendices. Appendix SA1 is devoted to the analysis of the procurement game under different specifications: The proofs are tedious but not novel. Appendix SA2 presents how production risk has been modelled in the French offshore auction application and additional assumptions made for the calibration exercise. Appendix SA3 is a complement to the contract design analysis.

Appendix SA1: Equilibrium analysis of the competitive procurement

Tie-breaking rule: In our analysis under full complete information, we assume that ties are broken in favor of the firm with a strictly positive payoff when it wins (if there is a single firm with a strictly positive payoff).[42] In the other cases under complete information, the tie-breaking rule can be random. Note that the case with more than one firm with a strictly positive payoff is not relevant to the equilibrium path since one of these firms would have incentives to lower its bid, which would violate the equilibrium conditions.
For Proposition 11, we make an additional technical assumption for the tie-breaking rule. Unlike the tie-breaking assumption above, this additional assumption only matters in non-generic subcases, and is introduced here for simplicity. We assume that if there is a tie where multiple firms have a zero payoff when winning, then the tie is broken in favor of the truthful firms.

Equilibrium analysis under full complete information

Let $k_i \in \{T, S\}$ denote the type of firm i regarding strategic/truthful behavior. In this subsection, we assume that the vector of types $(k_1, \cdots, k_N)$ is common knowledge among firms.
A general equilibrium characterization – Consider the lowest-price auction with a given menu of contracts $\mathcal{M}$. Let $\widehat{b}_i := \widehat{b}_i^{k_i}(\mathcal{M})$ denote the zero-profit bid of firm i.

Proposition 14. *Under full complete information, the contractor is a firm with the lowest zero-profit bid, and the equilibrium bid is the second lowest zero-profit bid. Formally, the winning firm* $i^w \in Arg\min_{i=1,\dots,N} \widehat{b}_i$ *and its equilibrium bid* b^w *is equal to* $\min_{i \neq i^w} \widehat{b}_i$. *Furthermore, the*

[42] More generally, see Simon and Zame (1990) on the need to endogenize the tie-breaking rule to guarantee equilibrium existence in discontinuous strategic games, and thus in particular in first-price auctions.

contractor i^w chooses a pair of project and declaration (f, σ) such that $f \in f^T(b^w, i^w)$ and $\sigma = \tilde{\sigma}(f^T(b^w, i^w))$ if $k_{i^w} = T$, and $(f, \sigma) \in Arg\max_{(f,\sigma)\in\mathcal{F}_{i^w}\times\Sigma} \Pi_{i^w}(b^w, f, \sigma)$ if $k_{i^w} = S$.

Proof of Proposition 14

First, consider pure strategy equilibria. Let b_i^{eq} denote the equilibrium bid of firm i and $b^{eq} := \min_{i=1,\dots,N} b_i^{eq}$ denote the minimum of the equilibrium bids. Our undominated strategy assumption implies that $b_i^{eq} \geq \widehat{b}_i$ for each i. If the equilibrium bid of a given firm i is strictly above b^{eq}, then its winning probability in the lowest-price auction is zero, and we must have $\widehat{b}_i \geq b^{eq}$. Otherwise, firm i would raise a strictly positive payoff by bidding a bit below b^{eq} and winning the auction for sure. In other words, firm i would strictly increase its payoff by deviating from its equilibrium bid, which would create a contradiction.

If there is a single firm bidding b^{eq} in equilibrium, then that firm would strictly increase its expected payoff by bidding a bit above b^{eq} while still winning the auction with probability one. Thus, there are at least two firms that bid exactly b^{eq}. In the corresponding tie, there is at most one firm that wins the auction with probability one. Consider a firm i that bids b^{eq} and wins the auction with probability strictly less than one. If $\widehat{b}_i < b^{eq} = b_i^{eq}$, then firm i would strictly increase its expected payoff by undercutting slightly its bids to win with probability one. We then have $\widehat{b}_i = b^{eq}$. Finally, we obtain that the tie is such that there is at most one firm $j \in \{1, \dots, N\}$ such that $\widehat{b}_j < b^{eq}$: this follows from the fact that we have shown that $\widehat{b}_i \geq b^{eq}$ if $b_i^{eq} > b^{eq}$, and that we cannot have two firms i and i' such that $\max\{\widehat{b}_i, \widehat{b}_{i'}\} < b_i^{eq} = b_{i'}^{eq} = b^{eq}$. Given our tie-breaking rule, if there is a firm j with $\widehat{b}_j < b^{eq}$, that firm will surely win the auction, and firms $i \neq j$ would have a zero-profit bid $\widehat{b}_i \geq b^{eq}$. Otherwise, if no such firm exists, we have $\widehat{b}_i = b^{eq}$ for the (at least two) firms i such that $b_i^{eq} = b^{eq}$, while $\widehat{b}_j > b^{eq}$ for the firms j bidding strictly above b^{eq}. In any case, we obtain that $i^w \in \operatorname{Arg\,min}_{i=1,\dots,N} \widehat{b}_i$ and $b_{i^w}^{eq} = b^{eq} = \min_{i \neq i^w} \widehat{b}_i$.

These are necessary conditions for any (pure strategy) equilibrium, and it is straightforward that such an equilibrium exists: e.g., where $b_i^{eq} = \widehat{b}_i$ if $i \neq i^w$ and $b_{i^w}^{eq} = \min_{i \neq i^w} \widehat{b}_i$.

We have then established Proposition 14 when restricting to pure strategy equilibria. It is left to the reader to show that in any mixed strategy equilibrium we still have $i^w \in \operatorname{Arg\,min}_{i=1,\dots,N} \widehat{b}_i$ and $b_{i^w}^{eq} = b^{eq} = \min_{i \neq i^w} \widehat{b}_i$. The main steps are as follows: If $\operatorname{Arg\,min}_{i=1,\dots,N} \widehat{b}_i$ is a singleton $\{i^*\}$, then in equilibrium this firm should win the auction for sure (we have $i^w = i^*$ with probability one), and i^w cannot bid strictly above $\min_{i \neq i^w} \widehat{b}_i$ otherwise a losing firm could win the auction by undercutting firm i^w's bid. The undominated strategy assumption and our tie-breaking rule assumption also guarantee that losing bidders would never bid strictly below $\min_{i \neq i^w} \widehat{b}_i$. Note that such mixed strategy equilibria do exist,[43] but the outcome does not differ

[43] For example, each bidder $i \neq i^*$ could mix uniformly on the interval $[\widehat{b}_i, \widehat{b}_i + \epsilon]$ while $b_{i^*}^{eq} = \min_{i \neq i^w} \widehat{b}_i$. If ϵ is small enough, then these strategies constitute an equilibrium.

from the one under pure strategy equilibria.
Otherwise, if $\operatorname{Arg\,min}_{i=1,\ldots,N} \widehat{b}_i$ is not a singleton, competition between the firms leads to zero profits (as in Bertrand competition with symmetric firms, which precludes any rent). The contractor then belongs to the set $\operatorname{Arg\,min}_{i=1,\ldots,N} \widehat{b}_i$, and bids $\min_{i=1,\ldots,N} \widehat{b}_i$. **Q.E.D.**

Application of Proposition 14 to menus of linear contracts:

Let us apply Proposition 14 to both the linear cash and the linear royalty lowest-price auctions, in order to formally derive some results presented in section 4.1. In these auctions, the menu of contracts does not depend on the declaration σ: strategic firms do not differ from truthful firms and $\widehat{b}_i^k$ does not depend on k. Then we will use the shortcut notation $\widehat{b}_i$ in the following. We also use the notation i^w for the winning firm, $i^w \in \operatorname{Arg\,min}_{i=1,\ldots,N} \widehat{b}_i$, and $b^{eq} := \min_{i \neq i^w} \widehat{b}_i$ for the corresponding equilibrium bid.

Linear cash lowest-price auctions:

In a linear cash lowest-price auction (characterized by the parameter λ), we have in general

$$\widehat{b}_i = \min\{b \in \mathbb{R} | \max_{f \in \mathcal{F}_i} \mathbb{E}_f[U_i(\lambda \cdot q + b - C_i(f))] \geq 0\}. \tag{18}$$

If firms are risk neutral, as we assume next, Equation (18) simplifies to $\widehat{b}_i = -\max_{f \in \mathcal{F}_i} \{\lambda \cdot \bar{q}_f - C_i(f)\}$. Let $f^w \in \operatorname{Arg\,max}_{f \in \mathcal{F}_{i^w}} \{\lambda \bar{q}_f - C_{i^w}(f)\}$ be the project chosen by the winning firm i^w in equilibrium. Note that the set of optimal projects for a firm does not depend on its bid.
Since $i^w \in \operatorname{Arg\,min}_{i=1,\ldots,N} \{\widehat{b}_i\}$, then we have $-\widehat{b}_{i^w} = \lambda \cdot \bar{q}_{f^w} - C_{i^w}(f^w) \geq -\widehat{b}_i \geq \lambda \cdot \bar{q}_f - C_i(f)$ for any pair (i, f). This further implies that the equilibrium social welfare $v \cdot \bar{q}_{f^w} - C_{i^w}(f^w) = \lambda \cdot \bar{q}_{f^w} - C_{i^w}(f^w) + (v - \lambda)\bar{q}_{f^w}$ which is greater than $\lambda \cdot \bar{q}_{f^*_{i^*}} - C_{i^*}(f^*_{i^*}) + (v - \lambda)\bar{q}_{f^w} = SW^* + (v - \lambda)[\bar{q}_{f^w} - \bar{q}_{f^*_{i^*}}]$, where the pair $(i^*, f^*_{i^*})$ corresponds to a welfare-optimal allocation. As noted in section 4.1, in the case $v = \lambda$, the pair (i^w, f^w) necessarily corresponds to a welfare-optimal allocation, and otherwise the difference between the equilibrium and the optimal social welfare is bounded by a term depending on the difference $(v - \lambda)$.

Linear royalty lowest-price auctions:

In a linear royalty auction (characterized by the parameter μ), we have

$$\widehat{b}_i = \min\{b \in \mathbb{R} | \max_{f \in \mathcal{F}_i} \{\mathbb{E}_f[U_i(b \cdot \bar{q}_f + \mu - C_i(f)\})] \geq 0\}. \tag{19}$$

We consider that firms are risk neutral, and we let $f^w \in \operatorname{Arg\,max}_{f \in \mathcal{F}_{i^w}} \{b^{eq} \cdot \bar{q}_f - C_{i^w}(f)\}$ be the project chosen by the winning firm in equilibrium. Note that the set of optimal projects for a firm now depends on its submitted bid, even if it is risk neutral.
From Proposition 14, the winning firm i^w raises a positive payoff if choosing project f^w at

equilibrium bid b^{eq}, while the other firms would raise a negative payoff with any possible project if winning at b^{eq}. Formally, the winning firm has a payoff $b^{eq} \cdot \bar{q}_{f^w} + \mu - C_{i^w}(f^w) \geq 0$, and for any pair (i, f) with $i \neq i^w$ we have $b^{eq} \cdot \bar{q}_f + \mu - C_i(f) \leq 0$. Finally, we get that $b^{eq} \cdot \bar{q}_{f^w} - C_{i^w}(f^w) \geq b^{eq} \cdot \bar{q}_f - C_i(f)$ for any pair (i, f). This further implies that the equilibrium social welfare $v \cdot \bar{q}_{f^w} - C_{i^w}(f^w) = b^{eq} \cdot \bar{q}_{f^w} - C_{i^w}(f^w) + (v - b^{eq}) \cdot \bar{q}_{f^w}$ is greater than $SW^* + (v - b^{eq})[\bar{q}_{f^w} - \bar{q}_{f^*_{i^*}}]$. As noted in section 4.1, the pair (i^w, f^w) corresponds to a welfare-optimal allocation in the case where $v = b^{eq}$ and the difference between the equilibrium and the optimal social welfare is bounded by a term depending on $(v - b^{eq})$.

Proof of Proposition 11

Each firm i is characterized by its type $k_i \in \{T, S\}$ regarding strategic/truthful behavior and its cost c_i. Firms are ranked such that $c_1 \leq \cdots \leq c_N$. From Proposition 14, in equilibrium, the contractor i^w belongs to the set $\operatorname{Arg\,min}_{i=1,\cdots,N} \widehat{b}_i^{k_i}$ and submits the bid $\tilde{b} = \min_{i \neq i^w} \widehat{b}_i^{k_i}$ (i.e. the second-lowest zero-profit bid) and, if strategic, reports a declaration $\sigma^* \in \operatorname{Arg\,max}_{\sigma \in \Sigma} \mathbb{E}_f[T_{(\tilde{b},\sigma)}(q)]$. Given that the menu $\mathcal{M}'$ is assumed to be manipulable for any bid and any firm, then we must have $\sigma^* \neq \tilde{(}f)$.
From our tie-breaking rule, if $\operatorname{Arg\,min}_{i=1,\cdots,N} b_i^{k_i}$ is not a singleton, then the contractor is strategic only if all firms in $\operatorname{Arg\,min}_{i=1,\cdots,N} b_i^{k_i}$ are strategic.

Under the strongly strategy-proof menu $\mathcal{M}$, the equilibrium bid (or second-lowest zero-profit bid) $b^{eq} = \widehat{b}_2(\mathcal{M})$, is characterized by $\mathbb{E}_f[T_{(\widehat{b}_2(\mathcal{M}),\tilde{\sigma}(f))}(q)] = c_2$, and the buyer's expected cost is then c_2.
Now consider the menu $\mathcal{M}'$ such that it is production-insuring relative to $\mathcal{M}$. Proposition 3 and the assumption that $\mathcal{M}'$ is manipulable for any i and any bid imply that for any firm i we have

$$\widehat{b}_1^S(\mathcal{M}') \leq \widehat{b}_i^S(\mathcal{M}') < \widehat{b}_i^T(\mathcal{M}') \quad \text{and} \quad \widehat{b}_1^S(\mathcal{M}') < \widehat{b}_1^T(\mathcal{M}') \leq \widehat{b}_i^T(\mathcal{M}'). \tag{20}$$

Below we drop the dependence on $\mathcal{M}'$ to alleviate the notation.
Case 1: $k_{i^w} = S$. Assuming that the winning firm i^w is strategic, consider a firm j with the second-lowest zero-profit bid, that is, $j \in \operatorname{Arg\,min}_{i \neq i^w} b_i^{k_i}$. Then consider two different cases: either j is strategic, i.e. $k_j = S$, or j is truthful, i.e. $k_j = T$.

Case 1a: $k_j = S$. From the definition of the zero-profit bid of a strategic firm, note that $\max_{\sigma \in \Sigma} \mathbb{E}_f[T_{(\widehat{b}_i^S,\sigma)}(q)] = c_i$ for both $i = i^w$ and $i = j$. Since we must have $\widehat{b}_{i^w}^S \leq \widehat{b}_j^S$, this implies that $i^w < j$ and thus that $j \geq 2$. Then the buyer's expected cost is $\max_{\sigma \in \Sigma} \mathbb{E}_f[T_{(\widehat{b}_j^S,\sigma)}(q)] = c_j \geq c_2$.

Case 1b: $k_j = T$. From the definition of the zero-profit bid of a truthful firm, we have $\mathbb{E}[T_{(\widehat{b}_j^T,\tilde{\sigma}(f))}(q)] = c_j$. The buyer's expected cost is equal to

$\max_{\sigma\in\Sigma}\mathbb{E}_f[T_{(\widehat{b}_j^T,\sigma)}(q)] > \mathbb{E}_f[T_{(\widehat{b}_j^T,\tilde{\sigma}(f))}(q)] = c_j$. Then, if $j \neq 1$, the buyer's expected cost is larger than c_2.
Now let us suppose that $j = 1$, which implies that $i^w \geq 2$. Then we have $\widehat{b}_{i^w}^S \leq \widehat{b}_1^T$, and therefore the strategic firm i^w makes a positive surplus at price b_1^T. Thus, the buyer's expected cost is greater than $c_{i^w} \geq c_2$.

If all firms are strategic, then we are in Case 1a with $i^w = 1$ and $j = 2$, so the buyer's expected cost is equal to c_2. On the other hand, if the contractor is the only strategic firm, then we must be in case 1b with either $i^w = 1$ and $j = 2$ or $i^w \geq 2$ and $j = 1$. If $i^w = 1$ and $j = 2$, then the buyer's expected cost is $\max_{\sigma\in\Sigma}\mathbb{E}_f[T_{(\widehat{b}_2^T,\sigma)}(q)] > \mathbb{E}_f[T_{(\widehat{b}_2^T,\tilde{\sigma}(f))}(q)] = c_2$. If $i^w \geq 2$ and $j = 1$, then the tie-breaking rule implies that $b_{i^w}^S < b_1^T$ (if $b_{i^w}^S = b_1^T$, then the tie should never be broken in favor of the strategic firm i^w) and thus that the strategic firm i^w makes a strictly positive surplus when winning at bid b_1^T, which implies that the buyer's expected cost is strictly greater than c_{i^w} and thus strictly greater than c_2.
Case 2: $k_{i^w} = T$**.** From (20), we must have $b_{i^w}^T = b_1^T$ or equivalently $c_{i^w} = c_1$, i.e. the contractor must be (one of) the most efficient firm(s). We can then assume without loss of generality that $i^w = 1$. The buyer's expected cost is then equal to $\mathbb{E}[T_{(\tilde{b},\tilde{\sigma}(f))}(q)]$ where
$\tilde{b} = \min_{i=2,\cdots,N}\{\widehat{b}_i^{k_i}\} \leq b_2^T$. The buyer's is then less than $\mathbb{E}[T_{(b_2^T,\tilde{\sigma}(f))}(q)] = c_2$.
If all firms are truthful, then $\tilde{b} = b_2^T$ and the buyer's expected cost is equal to c_2. Conversely, if firm 2 is strategic, then $\tilde{b} = b_2^S < b_2^T$ and the buyer's expected cost is strictly smaller than c_2.
Q.E.D.

Equilibrium analysis under incomplete information with payoff-symmetric firms (for Proposition 12)

Throughout this subsection we assume that ties are broken randomly. Let us characterize the equilibrium outcome when firms are payoff-symmetric such that the zero-profit bids $\widehat{b}_i^k$, $k \in \{T, S\}$, and the sets $f_i^T(b)$ and $Q_i^*(b)$ do not depend on i and are then denoted by $\widehat{b}^k$, $f^T(b)$ and $Q^*(b)$. The equilibrium analysis depends on the strategic/truthful types k of the firms and their beliefs about the types of their competitors. We assume that it is common knowledge among the firms that they are payoff symmetric.
The types of firms, and thus the number of strategic firms, are random: we consider an incomplete information symmetric setting where each firm is strategic (resp. truthful) with probability α (resp. $1-\alpha$) independently of the others. Each firm knows its own type and the parameter $\alpha \in]0,1[$, but ignores the types of the other firms.

Proposition 15. *Consider payoff-symmetric firms in a procurement that is manipulable at the zero-profit bid $\widehat{b}^T$ of a truthful firm. Suppose that each firm is strategic with probability $\alpha \in]0,1[$.*

Consider a menu that is manipulable for any bid (i.e. $\Pi^S(b) > \Pi^T(b)$ *for any bid* b*). In equilibrium, all firms adopt the following strategy:*

- *If the firm is truthful, it makes a decision* $(\widehat{b}^T, f, \tilde{\sigma}(f))$ *with* $f \in f^T(\widehat{b}^T)$*.*
- *If the firm is strategic, it adopts a mixed strategy, consisting of the decision* $(b, f(b), \sigma(b))$ *with* $(f(b), \sigma(b)) \in Q^*(b)$*, where the bid* b *is distributed according to the CDF*

$$G(b) = \max\{1 - \frac{1-\alpha}{\alpha}\left(\sqrt[N-1]{\frac{\Pi^S(\widehat{b}^T)}{\Pi^S(b)}} - 1\right), 0\}.$$

The upper (resp. lower) bound of the distribution G *is equal to* $\widehat{b}^T$ *(strictly greater than* $\widehat{b}^S$*).*

From Proposition 15 we get the analog of Corollary 4 with the twist that the bid distribution is now stochastic: any equilibrium bid realization of a menu of contract $\mathcal{M}'$ that provides more insurance than a strongly strategy-proof menu $\mathcal{M}$ is lower than the equilibrium bid under $\mathcal{M}$. We also see that as the probability of a firm being strategic increases, the bid distribution shifts down. However, lower bids do not imply a higher payoff for the buyer. On the contrary, strategic firms reap the positive payoff $(1-\alpha)^{N-1}\Pi^S(\widehat{b}^T) > 0$. Each firm is strategic with probability α and thus the joints rents captured by the N firms are equal to $N \cdot \alpha \cdot (1-\alpha)^{N-1}\Pi^S(\widehat{b}^T) > 0$. Under risk neutrality, the buyer's expected cost is equal to the production cost c plus the joints rents captured by the firms, which concludes the proof of Proposition 12.

Remarks: Our equilibrium analysis is analogous to the analysis of first-price auctions with two (possibly risk-averse) symmetric bidders with binary valuations developed by Maskin and Riley (1985): being strategic (resp. truthful) in our procurement setting corresponds to having a high (resp. low) valuation in Maskin and Riley's (1985) auction setting.[44] However, there are three differences: First we consider any number of bidders.[45] Second, the expected payoff of a firm $\Pi^k(b)$, $k \in \{T, S\}$ is no longer linear in b. Finally, we consider procurements involving a (secret) project choice and a (public) declaration about it. If the sets $f^T(\widehat{b}^T)$ and $Q^*(b)$ are not singletons for any b in the support of G, this extra dimension leads to equilibrium multiplicity (in contrast to the analysis of Maskin and Riley (1985)). However, all equilibria are somehow equivalent in that they involve the same bid distribution and the same expected payoffs for the firms. In general, however, the buyer is not indifferent between them.

Proof of Proposition 15

[44] Doni and Menicucci (2012) extend the analysis to two asymmetric bidders when bidders are assumed to be risk neutral.

[45] Technically, this is the most challenging extension because it is less straightforward to show that all bidders use the same bid distribution when we have more than two bidders.

The equilibrium analysis is analogous to Maskin and Riley (1985): having a low (resp. high) valuation here corresponds to being a truthful (resp. strategic) firm. Note that the assumption that the procurement is manipulable at the bid $\widehat{b}^T$ implies that $\Pi^S(\widehat{b}^T) > \Pi^T(\widehat{b}^T) = 0$, which guarantees that strategic firms make positive surplus. As in Maskin and Riley (1985), we have in equilibrium that truthful bidders must submit their zero-profit bid $\widehat{b}^T$ and that strategic bidders reap a strictly positive payoff and bid below $\widehat{b}^T$. We do not detail the corresponding arguments because they are identical in the present setting.

Let us focus on the bidding strategies of strategic bidders, which are denoted next by the CDFs G_i, $i = 1, \dots, N$. Let $Sup(G_i) \subseteq]-\infty, \widehat{b}^T]$ denote the support of the distribution G_i. We necessarily have $Sup(G_i) \subseteq \bigcup_{j \neq i} Sup(G_j) \cup \{\widehat{b}^T\}$ for any i: any bid b in the support of firm i's bid distribution must belong to the support of the highest bid among its competitors. Otherwise firm i, if strategic, would strictly benefit from submitting a bid slightly above b, since it would not change its (positive) probability of winning and would strictly increase its payoff conditional on winning.

Let us show that the distribution G_i has no atom. Suppose b^* is an atom of G_i. We cannot have $b^* = \widehat{b}^T$ in equilibrium, because it would raise a contradiction with strategic bidders getting a strictly positive payoff: bidder i would benefit by bidding slightly below $\widehat{b}^T$, so that it wins the auction with a strictly greater probability. Now consider $b^* < \widehat{b}^T$ so that $b^* \in \bigcup_{j \neq i} Sup(G_j)$. Let $G^*_{-i} := 1 - \prod_{j \neq i}(1 - \alpha + \alpha(1 - G_j))$ denote the distribution of the lowest bid among firm i's competitors. First b^* cannot be an atom of G^*_{-i}: if it were, a strategic firm i would benefit from bidding a bit below b^* to win with probability one. Second, consider the case where G^*_{-i} has no mass in the right neighborhood of b^*: then a strategic firm i would benefit from bidding a bit above b^*: it would not change its (positive) probability of winning and would strictly increase its payoff conditional on winning. The remaining case (given that $b^* \in \bigcup_{j \neq i} Sup(G_j)$) is when G^*_{-i} has some mass on the right neighborhood of b^*. In such a case, there exist $j \neq i$ such that G_j has some mass on the right neighborhood of b^*. Then there exists a strategic firm j that would benefit from bidding slightly below b^* instead of slightly above b^*: this follows from the discontinuity of the winning probability around b^* given the atom while the function $b \mapsto \Pi^S(b)$ is continuous. Thus, we have shown that the distribution G_i, $i = 1, \dots, N$ has no atom. Let g_i denote the corresponding density (which may be discontinuous).

Let $\pi_i(b)$ denote the expected payoff of a strategic firm i bidding b. Since the bidding distributions of strategic firms are atomless, the probability of facing a tie with a bid below $\widehat{b}^T$ is zero, and thus we have

$$\pi_i(b) = \prod_{j \neq i} [1 - \alpha + \alpha(1 - G_j(b))] \cdot \Pi^S(b)$$

for any $b < \widehat{b}^T$. By continuity, we let $\pi_i(\widehat{b}^T) := (1 - \alpha)^{N-1} \cdot \Pi^S(\widehat{b}^T)$, which corresponds to the

expected payoff of a strategic firm bidding $\widehat{b}^T$ when ties are broken in its favor.
The equilibrium condition for any strategic firm i implies an indifference condition for any bid on its support: $\pi_i(b)$ does not depend on b on the set $Sup(G_i)$. If $\widehat{b}^T \in Sup(G_i)$, we have for any $b \in Sup(G_i)$:

$$\prod_{j \neq i}[1-\alpha+\alpha(1-G_j(b))] \cdot \Pi^S(b) = (1-\alpha)^{N-1} \cdot \Pi^S(\widehat{b}^T) \tag{21}$$

Let $\underline{b}_i$ (resp. $\bar{b}_i \leq \widehat{b}^T$) denote the lower (resp. upper) bound of $Sup(G_i)$. Let $\bar{b} \equiv max_{i=1,\dots,N}\{\bar{b}_i\}$. If $\bar{b} < \widehat{b}^T$, then a firm $j \in \operatorname{Arg\,max}_{i=1,\dots,N}\{\bar{b}_i\}$ would strictly benefit from bidding slightly above $\bar{b} = \bar{b}_j$ (while staying below $\widehat{b}^T$) instead of bidding $\bar{b}_j \in Sup(G_j)$. So there is at least one firm i_1 such that $\bar{b}_{i_1} = \widehat{b}^T$: there is a positive mass of bids in the left neighborhood of $\widehat{b}^T$. Since $Sup(G_{i_1}) \subseteq \bigcup_{j \neq i} Sup(G_j) \cup \widehat{b}^T$, we finally obtain that there exists a firm i_2 such that $\bar{b}_{i_2} = \widehat{b}^T$. By multiplying both sides of the indifference condition (21) by $[1-\alpha+\alpha(1-G_i(b))]$, we obtain for $i \in \{i_1, i_2\}$ that

$$G_i(b) = \frac{1}{\alpha} \cdot [1 - \frac{\prod_{j=1}^{N}[1-\alpha+\alpha(1-G_j(b))] \cdot \Pi^S(b)}{(1-\alpha)^{N-1} \cdot \Pi^S(\widehat{b}^T)}] = \frac{1}{\alpha} \cdot [1 - \frac{G^*(b) \cdot \Pi^S(b)}{(1-\alpha)^{N-1} \cdot \Pi^S(\widehat{b}^T)}] \tag{22}$$

for any $b \in Sup(G_i)$ and where $G^*(b) := \prod_{j=1}^{N}[1-\alpha+\alpha(1-G_j(b))]$. Note that the right-hand term in (22) does not depend on i.
Let $N_1 := \{i \in \{1,\dots,N\} | \bar{b}_i = \widehat{b}^T\}$. If $N_1 \neq \{1,\dots,N\}$, let $\bar{\bar{b}} := \max_{j \notin N_1} \bar{b}_j < \bar{b}$. If $N_1 = \{1,\dots,N\}$, let $\bar{\bar{b}} := \min_{i \in N_1}\{\underline{b}_i\}$. Let us show by contradiction that $[\bar{\bar{b}}, \bar{b}] \subseteq \bigcup_{i \in N_1} Sup(G_i)$. In particular, it is equivalent to saying that there is no gap in the support of the lowest bid when $N_1 = \{1,\dots,N\}$. If there exist $b^* \in]\bar{\bar{b}}, \bar{b}[$ such that $b \notin Sup(G_i)$ for any i, then if we take the upper bound of the set $\{b \in Sup(G^*) | b < b^*\}$, denoted next by $\tilde{b} < b^*$, then a strategic firm bidding $\tilde{b}$ in equilibrium would strictly benefit from bidding b^* which raises a contradiction.
Let us define the function $\tilde{G}$ on $[\bar{\bar{b}}, \bar{b}]$ such that $\tilde{G}(b) := \frac{1}{\alpha} \cdot [1 - \frac{G^*(b) \cdot \Pi^S(b)}{(1-\alpha)^{N-1} \cdot \Pi^S(\widehat{b}^T)}]$. Since $[\bar{\bar{b}}, \bar{b}] \subseteq \bigcup_{i \in N_1} Sup(G_i)$ and $\tilde{G}$ is continuous, we obtain that the function $\tilde{G}$ is non-decreasing. Furthermore, $\tilde{G}$ cannot be flat on a given interval $I \subseteq [\bar{\bar{b}}, \bar{b}]$, because it would imply that G^* has no mass on this interval I which would raise a contradiction with $[\bar{\bar{b}}, \bar{b}] \subseteq \bigcup_{i \in N_1} Sup(G_i)$. So the function $\tilde{G}$ must be increasing on $[\bar{\bar{b}}, \bar{b}]$. Since $G_i(b) = \tilde{G}(b)$ on $Sup(G_i)$ for every i, we finally get $G_i(b) = \tilde{G}(b)$ for any $b \in [\bar{\bar{b}}, \bar{b}]$ and $i \in N_1$.
If $N_1 = \{1,\dots,N\}$, then any equilibrium must be symmetric ($G_i = G$ for any i) and, given the indifference condition, is characterized by

$$G(p) = 1 - \frac{1-\alpha}{\alpha}\left(\sqrt[N-1]{\frac{\Pi^S(\widehat{b}^T)}{\Pi^S(b)}} - 1\right) \tag{23}$$

on the interval $[\underline{b}^*, \widehat{b}^T]$, where $\underline{b}^*$ is the unique solution of $[1-\alpha+\alpha(1-G(\underline{b}^*))]^{N-1} \cdot \Pi^S(\underline{b}^*) = (1-\alpha)^{N-1} \cdot \Pi^S(\widehat{b}^T)$. It is then straightforward that strategic firms would not benefit by bidding outside the interval $[\underline{b}^*, \widehat{b}^T]$.
The rest of the proof consists in showing that we must have $N_1 = \{1, \dots, N\}$. Suppose, on the contrary, that $N_1 \neq \{1, \dots, N\}$. Let $N_2 := \{i | \bar{b}_i = \bar{\bar{b}}\} \neq \emptyset$ where $\bar{\bar{b}}$ is defined as above. On the interval $[\bar{\bar{b}}, \widehat{b}^T]$, the equilibrium strategies of the firms in N_1 are given by (23). Consider two different cases regarding $\bar{\bar{b}}$ and $\underline{b}^*$. A) If $\bar{\bar{b}} < \underline{b}^*$, there would be a gap in the bid distribution, which would raise a contradiction (firms in N_2 would benefit by bidding slightly above $\bar{\bar{b}}$ while staying below $\underline{b}^*$). B) Now consider that $\bar{\bar{b}} \geq \underline{b}^*$, i.e., the case where the bid $\bar{\bar{b}}$ belongs to the support of the bid distribution of firms in both N_1 and N_2. Note that by construction of $\bar{\bar{b}}$, we have $G_i(\bar{\bar{b}}) = 1$ if $i \notin N_1$ and $G_i(\bar{\bar{b}}) < 1$ for $i \in N_1$. However, given (22), at bid $\bar{\bar{b}}$, $G_i(\bar{\bar{b}})$ should be equal for firms i in $N_1 \cup N_2$. So we have raised a contradiction and thus shown that $N_1 = \{1, \dots, N\}$, which concludes the proof.
Q.E.D.

On the optimality of the LCMR procurement under endogenous entry (for Proposition 5)

In our setting with exogenous entry and when there is at least to firms ($N \geq 2$) with $SW_i^* > SW^{NO}$ for each firm i, there is no need to specify a reserve price under the LCMR procurement: it is always optimal to develop a project not only in the optimal allocation with all firms but also if we exclude the optimal firm. More generally, the LCMR procurement should involve a reserve price equal to $-SW^{NO}$. When there is a single firm, we consider that the procurement involves a reserve price equal to $-SW^{NO}$ such that in equilibrium the contract will be the $(\bar{p}, -SW^{NO})$-linear contract and the payoff of contractor, say i, is equal to $SW_i^* - SW^{NO}$. Throughout this section, we assume that firms are risk neutral: without loss of generality, it corresponds to $U_i(x) = x$ in our model. Consider furthermore that firms learn their cost function after deciding to enter the procurement which involves a sunk cost $c_e > 0$ which is assumed to be common to all firms. The cost functions C_i, $i = 1, \dots, N$, are assumed to be conditionally i.i.d. random variables. The N firms are thus ex ante symmetric: the distribution of the cost function C_i is the same for each firm i and does not depend on entry decisions. We emphasize that we allow cost functions to be correlated across firms (e.g. environments where all firms have the same cost functions corresponds to a special case). The entry stage, which takes place between stages 1 and 2, is assumed to be uncoordinated: ex ante symmetric firms decide simultaneously whether to enter or not the procurement. As in Levin and Smith (1994) and Jehiel and Lamy (2015), we restrict attention to symmetric equilibria at the entry stage and we assume that c_e is large enough such that full entry is not an equilibrium under the LCMR procurement (or equivalently is not socially efficient under an ex post efficient procurement as explained below). After entry, we assume that the cost functions of the n entrants, $C_1, \ldots, C_n$, are common knowledge among entrants. Let π denote the probability that each firm enters the procurement. Given that entry represents a physical sunk cost (and not a transfer between the buyer and the bidders as in auctions with entry fees), the social welfare is now equal to $\bar{p}\bar{q}_f - C_i(f) - nc_e$ if there are n entrants and if the contractor is firm i with the cost function C_i choosing project f. For a given procurement, let $SW[n]$ denote the expected gross social welfare (absent of the entry costs) when there are n entrants. In particular, if $n = 0$, we have $SW[0] = SW^{NO}$. If we consider a procurement where there is always a winning firm denoted i^w and a winning project f^w if $n \geq 1$, then we would have $SW[n] = \mathbb{E}[\bar{p} \cdot \bar{q}_f^w - C_{i^w}(f^w)|n]$. Note that the expectation is over the realization of the cost functions $C_1, \ldots, C_n$ which also drives the determination of i^w and p^w. Under the entry probability π, the expected total social welfare $TSW(\pi)$ is then equal to $\sum_{n=0}^{N} \{\frac{N!}{n!(N-n)!}\pi^n(1-\pi)^{N-n}SW[n] - nc_e\} = \sum_{n=0}^{N} \{\frac{N!}{n!(N-n)!}\pi^n(1-\pi)^{N-n}SW[n]\} - \pi N c_e$. Let π^{eq} the equilibrium entry probability. If we have $\pi^{eq} \in (0, 1)$, then firms are indifferent between entering or not the procurement and the expected profit of an entrant in the procurement (i.e. absent of the sunk entry cost) is equal to c_e. If $\pi^{eq} \in [0, 1)$, then firms' expected joint profit is null. On the contrary, if $\pi^{eq} = 1$, then the expected profit of all the firms in the procurement is greater or equal to $N \cdot c_e$. It implies that the buyer's expected payoff in equilibrium is lower than $TSW(\pi^{eq})$ and that both are equal if the entry probability $\pi^{eq} \in [0, 1)$. An upper bound for the expected total welfare is when the procurement selects ex post the socially optimal firm and the optimal project and also the optimal entry probability. Let $SW^*[n]$ denote the expected gross social welfare (i.e. excluding the entry costs) when there are n entrants under the ex post optimal allocation, i.e. when $i^w \in \operatorname{Arg\,max}_{i=1,\cdots,n} \max_{f\in\mathcal{F}} \bar{p}\bar{q}_f - C_i(f)$ and $f^w \in \operatorname{Arg\,max}_{f\in\mathcal{F}} \bar{p}\bar{q}_f - C_{i^w}(f)$ if $\max_{i=1,\cdots,n} \max_{f\in\mathcal{F}} \bar{p}\bar{q}_f - C_i(f) \geq SW^{NO}$ and no contract is signed otherwise. Let

$$TSW^*(\pi) := \sum_{n=0}^{N} \{\frac{N!}{n!(N-n)!}\pi^n(1-\pi)^{N-n}SW^*[n] - nc_e\}$$

denote the expected net welfare (i.e. including the entry costs) under an ex post efficient procurement when the entry probability is π. Naturally, in any procurement, we have $TWS(\pi) \leq TSW^*(\pi)$ for any entry probability $\pi \in [0, 1]$. Note that $\frac{TSW^*}{d\pi}(\pi) = N \cdot \left(\sum_{n=0}^{N-1} \frac{N-1!}{(n-1)!(N-1-n)!}\pi^n(1-\pi)^{N-1-n}\Big(SW^*[n+1] - SW^*[n]\Big) - c_e\right)$. As detailed in Jehiel and Lamy (2015), the differences $SW^*[n+1] - SW^*[n]$, $n = 0, \ldots, N-1$, are non-increasing in n with $SW^*[2] - SW^*[1] < SW^*[1] - SW^*[0]$ which implies that the function $TSW^*(.)$ is strictly concave on $[0, 1]$. Let π^* denote then the optimal entry probability.

If $SW^*[1]-SW^*[0]\leq c_e$, then $\frac{dTSW^*}{d\pi}(0)\leq 0$ and $\pi^*=0$. If $SW^*[N]-SW^*[N-1]\geq c_e$, then $\frac{dTSW^*}{d\mu}(1)\geq 0$ and $\pi^*=1$. If $SW^*[1]-SW^*[0]>c_e>SW^*[N]-SW^*[N-1]$, then $\pi^*\in(0,1)$ and π^* is characterized as the (unique) solution of the first order equation $\frac{dTSW^*}{d\mu}(\pi)(\pi^*)=0$ or equivalently

$$\sum_{n=1}^{N}\frac{N!}{n!(N-n)!}\pi^{n-1}(1-\pi)^{N-n}\Big(SW^*[n]-SW^*[n-1]\Big)=Nc_e. \tag{24}$$

The case where $SW^*[N]-SW^*[N_1]<c_e$, or equivalently $\pi^*\in[0,1)$, is referred to next as the case where the optimal entry profile is mixed. Since
$SW^*[N]-SW^*[N_1]\leq N\cdot(SW^*[1]-SW^*[0])$, then we have $SW^*[N]-SW^*[N_1]<c_e$ if N is large.
Since the buyer's expected cost in equilibrium is smaller than $TW(\pi^{eq})$ which is smaller than $TW^*(\pi)\leq TW^*(\pi^*)$, then $TSW^*(\pi^*)$ corresponds to an upper bound for the buyer's expected cost. Let us now establish that the LCMR procurement reaches this bound if the optimal entry profile is mixed.
If the procurement implements the optimal allocation and provides marginal rewards to the contractor (among $n\geq 1$ entrants), the expected gross payoff of the contractor is equal to $SW^*[n]-SW^*[n-1]$ and the equilibrium profile π^{eq} is then characterized by the condition

$$\sum_{n=1}^{N}\frac{N!}{n!(N-n)!}(\pi^{eq})^n(1-\pi^{eq})^{N-n}[SW^*[n]-SW^*[n-1]]=\pi^{eq}\cdot Nc_e \tag{25}$$

if this equation has a solution in $[0,1]$, while $\pi^{eq}=0$ (resp. $\pi^{eq}=1$) if $SW^*[1]-SW^*[0]<c_e$ (resp. $SW^*[N]-SW^*[N-1]>Nc_e$).
The optimality conditions that characterize π^* and the equilibrium conditions that characterizes π^{eq} in the LCMR procurement coincides and we obtain thus that $\pi^{eq}=\pi^*$: in equilibrium the entry profile is the welfare maximizing one under the LCMR procurement.
Finally, it implies that the buyer's expected payoff reaches the upper bound $TSW^*(\pi^*)$ under the LCMR procurement if $\pi^*<1$.
In words, our analysis with endogenous entry can be summarized shortly as follows:

Proposition 16. *Under complete information after entry and if the optimal entry profile is mixed, then the LCMR procurement maximizes both the social welfare and the buyer's expected payoff if firms are risk neutral.*

The procurement where each firm i submits a bid p_i (while project bids play no role), the firm i^w with the lowest bid wins the auction and the contract is the $(\bar{p},p_{i^{sl}})$-linear contract, where $p_{i^{sh}}$ corresponds to the second lowest bid (formally $p_{i^{sl}}=\min\{-SW^{NO},\min_{i\neq i^w}p_i\}$), is referred to next as the Second Lowest Price Marginal Rewards (SLCMR) procurement. If firms are risk neutral, then the project chosen by a firm for a given $(\bar{p}, \mu)$-linear contract does not depend on the lump-sum transfer μ. In the SLCMR procurement, it is a dominant strategy for each firm i to bid $-SW_i^*$ and the SLCMR procurement implements the optimal allocation and provides marginal rewards. The SLCMR procurement maximizes the social welfare and the buyer's expected cost without any restriction on firms' beliefs at the auction stage.

Remarks: 1) Levin and Smith (1994) consider a model with adverse selection but no moral hazard. In Levin and Smith's (1994) auction setting, the second price auction provides marginal rewards and solves both the adverse selection and the entry model. In our setting with moral hazard, providing marginal rewards solves also the moral hazard problem. 2) Jehiel and Lamy (2015) have established that Levin and Smith's (1994) analysis extend to environments with some kinds of ex ante asymmetries. The same arguments would work with moral hazard as well.

Appendix SA2: Details on the calibrations for the French offshore procurement auctions

The French government organized five separate procurement auctions to develop offshore wind farms in specific location she has selected, and with a given power capacity (i.e. the maximum power that the installation can produce, which is a verifiable technical feature).[46] The main characteristics of those projects (name, location, capacity in MW) are listed in Table 1.
In each auction, each bidder submits a bid and a reference production and the winning bidder is the firm submitting the lowest bid. When reporting the reference production, bidders were invited to report their expected yearly production. From a practical perspective, there were no explicit ranges for eligible reference production but unrealistic reference production would have lead to disqualification. Our analysis leaves out the disqualification risk associated with misreporting insofar as the optimal overestimation never exceeds 13% according to our estimation, a figure which is of the same order of magnitude as the prediction bias observed in practice for wind farms (Lee and Fields, 2020).

Table 1: Characteristics on the wind farm projects (source : European Commission (2019) and French Energy Regulatory Commission (2011, 2013)

Site	Location (lat.,long.)	Capacity in MW	IC (CAPEX) M €	OC (OPEX/year) M €	Awarded price €/MWh
Le Tréport	(50.1, 1.1)	496	2000	105	131
Ile d'Yeu	(46.9, -2.5)	496	1860	110	137
Fécamp	(49.9, 0.2)	497	1850	75	135.2
Courceulles	(49.5, -0.5)	448	1600	69	138.7
Saint-Nazaire	(47.2, -2.6)	496	1800	78	143.6

At the end of each year and during 20 years, the contractor receives the payment $b \cdot R_{q_0}(q)$ if his yearly production is q and where b and q_0 is the bid and reference production submitted in the auction. The function $R_{q_0}(q)$ is depicted in Figure **??**. Another mild difference with our static theoretical framework is that for each project f, we consider both a (fixed) investment cost IC_f occurring before production and (fixed) operating costs OC_f occurring each year. For a given decision (b, f, q_0), firms' expected payoff difference between winning and losing the auction can

[46] The auction and contract rules are provided (in French) by the French Energy Regulatory Commission for both auction rounds from 2011 and 2013: www.cre.fr/Documents/Appels-d-offres/Appel-d-offres-portant-sur-des-installations-eoliennes-de-production-d-electricite-en-mer-en-France-metropolitaine and www.cre.fr/Documents/Appels-d-offres/Appel-d-offres-portant-sur-des-installations-eoliennes-de-production-d-electricite-en-mer-en-France-metropolitaine2.

then be expressed as:

$$\mathbb{E}_f\left[U(\sum_{t=1}^{20}\frac{[b\cdot R_{q_0}(q_t)-OC_f]}{(1+r)^t})\right]-U(IC_f), \tag{26}$$

where the expectation is made with respect to the vector of yearly production $(q_1,\dots,q_{20})$, where we take the CRRA utiity function $U(x)=\frac{x^{1-\gamma}}{1-\gamma}$ and where r denotes firms' annual discount rate which is set equal to 5.7%.[47] For a given bid b and a given project f, a strategic firm reports an expected production $q_0^*(b,f)$ that maximizes the expression in (26).
Our cost assumptions for the various projects come from a report of the European Commission.[48] are reported in Table 1. Firms are assumed to be payoff-symmetric and to enter the procurement with a single project whose production distribution is build as follows. Hourly electricity productions of these farms are simulated using the model for 19 years (from 2000 to 2018) developed by Staffell and Pfenninger (2016) and this thanks to the website https://www.renewables.ninja/ to which the location and the characteristics of the turbine have been given as inputs. We take the Vestas V164 8000 turbine. Hourly production obtained from the simulator is then aggregated at the quarterly level. Then we bootstrap our 19 years of aggregated quarterly data to generate the distribution of yearly production: quarters are randomly drawn and summed to generate yearly production points. This resampling approach to generate more than our 19 original years of production is relevant if there is no significant autocorrelation between quarterly aggregate production.
Finally, the distribution of the vector of yearly-production $(q_1,\dots,q_{20})$ is build in the following way: each yearly-production q_t is the product of a yearly-dependent production drawn independently across years according to the bootstrapped distribution defined above with $1+\epsilon$ where ϵ is a non-year-dependant noise distributed according to a centered normal distribution with the variance σ^2. We assume that $\sigma=6.3\%$, which matches a mean absolute error of 5%. The noise ϵ, which reflects that firms do not have a perfect knowledge on their expected production,[49] is actually the main driver for the risk premiums relative to net present value of the subsidy contracts: contrary to weather risk, this additional risk is not averaged out over the 20 years of production.

[47] Our choice is based on an estimation of the cost of capital for onshore wind projects in France made by Angelopoulos et al. (2016).

[48] https://ec.europa.eu/competition/state_aid/cases1/201933/265141_2088479_221_2.pdf

[49] In the past, estimations of wind turbine expected production has suffered from important bias as surveyed by Lee and Fields (2020). The methodologies have been improved with the aim to reduce bias, but they still involve economically relevant errors: e.g. Jourdier and Drobinski (2017) show that the commonly used statistical model based on Weibull distributions lead to a mean absolute error around 4 or 5% of the average electricity production.

Appendix SA3: Complement to the contract design analysis

Proof of the results at the end of Section 4.3 for a parametric class of contracts.

As a complement to Theorem 10, we further study a much more restricted setting to provide some insights about how a risk-averse contractor reports its expected production depending on various parameters. The setting considered is as follows:

- The menu of contracts $\mathcal{M}^w := \{T^w_{(b,q_0)}\}_{(b,q_0)\in\mathbb{R}^2_+}$ is parameterized by $w \in]0,1[$ and is such that $T^w_{(b,q_0)}(q) = \lambda(b)\cdot q_0 + \mu(b)$ if $q \in [(1-w)q_0, (1+w)q_0]$ and $R_w(q,q_0) = \lambda(b)\cdot q + \mu(b)$ otherwise. In other words, the contractor is perfectly insured and its remuneration depends only on reported expected production q_0 as long as its actual production is no more than $\pm w\%$ away from q_0. Beyond this interval, the remuneration is the same as under the corresponding linear contract.

- The production risk is distributed according to $f \in \mathcal{F}_{sym}$ and its support is assumed to be equal to $[(1-\delta)\bar{q}, (1+\delta)\bar{q}]$ with $\delta \leq w$. A direct consequence of this last restriction is that a truthful contractor would be fully insured: the whole support of its production distribution is included in the area where the payment does not depend on q.

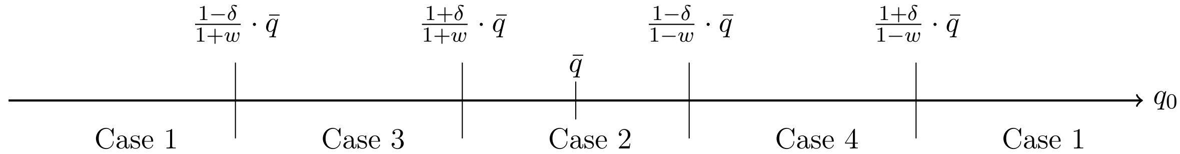
From now on, we fix the bid b and the distribution f. Let us then define the function $\bar{U} : \mathbb{R}_+ \mapsto \mathbb{R}$ such that $\bar{U}(x) := U_i(\lambda(b)\cdot x + \mu(b) - C_i(f))$ for a given contractor i. $\bar{U}$ is concave as U_i is concave. The contractor's payoff when reporting q_0 is equal to $\mathbb{E}_f[U(T^w_{(b,q_0)}(q) - C_i(f)]$. The contractor's payoff under truthful reporting is then equal to $\bar{U}(\bar{q})$ since the payment is always equal to $\lambda(b)\bar{q} + \mu(b)$ for any realization on the support of f. To derive the optimal reporting of q_0, we consider the contractor's payoff in four separate cases regarding the chosen q_0 which cover all possible reported q_0 (given the assumption $\delta \leq w$):

1. q_0 is such that actual production never falls in the insured range;

2. q_0 is such that actual production always falls in the insured range (which includes the case $q_0 = \bar{q}$, i.e. truthful reporting);

3. q_0 is such that actual production sometimes falls in the insured range, sometimes above;

4. q_0 is such that actual production sometimes falls in the insured range, sometimes below.



Case 1 Actual production never falls in the insured range if q_0 is chosen such that either $(1+\delta)\bar{q} < (1-w)q_0$ or $(1-\delta)\bar{q} > (1+w)q_0$, i.e., for any q_0 outside the interval $[\frac{1-\delta}{1+w}\bar{q}, \frac{1+\delta}{1-w}\bar{q}]$. For such q_0, the contractor's expected payoff is $\mathbb{E}_f[\bar{U}(q)] \leq \bar{U}(\mathbb{E}_f[q]) = \bar{U}(\bar{q})$. The last inequality results from the concavity of $\bar{U}$ and implies that the case 1 never brings a strictly better payoff to the contractor than truthful reporting.

Case 2 Actual production always fall in the insured range if q_0 is chosen such that $(1-w)q_0 \leq (1-\delta)\bar{q}$ and $(1+w)q_0 \geq (1+\delta)\bar{q}$, i.e., for $q_0 \in [\frac{1+\delta}{1+w}\bar{q}, \frac{1-\delta}{1-w}\bar{q}]$. In this interval, the contractor's expected payoff is $\mathbb{E}_f[\bar{U}(q_0)]$, which is then maximized for the highest value of q_0 within this interval, i.e. for $q_0 = \frac{1-\delta}{1-w}\bar{q} \geq \bar{q}$.

Case 3 This case corresponds to the reference productions such that the upper bound of the insurance range is within the support of f: $(1-\delta)\bar{q} < (1+w)q_0 < (1+\delta)\bar{q}$, or equivalently $q_0 \in]\frac{1-\delta}{1+w}\bar{q}, \frac{1+\delta}{1+w}\bar{q}[$. The contractor's expected payoff can then be expressed as

$$\mathbb{E}_f[U(T^w_{(b,q_0)}(q) - C_i(f))] = F((1+w)q_0) \cdot \bar{U}(q_0) + \int_{(1+w)q_0}^{(1+\delta)\bar{q}} \bar{U}(q)dF(q).$$

Let us define the distribution f^* from the (atomless) CDF f, by replacing the smooth part on the interval $[(1-\delta)\bar{q}, (1+w)q_0]$ by an atom at q_0. Formally, $F^*(q) = 0$ for $q < q_0$, $F^*(q) = F((1+w)q_0)$ for $q \in [q_0, (1+w)q_0]$ and $F^*(q) = F(q)$ for $q \geq (1+w)q_0$. Equipped with this definition we have $\Pi_i(b, f, q_0) = \mathbb{E}_{f^*}[\bar{U}(q)] \leq \bar{U}(\mathbb{E}_{f^*}[q])$ where the latter inequality comes from the concavity of $\bar{U}$. Therefore if we show that $\bar{U}(\mathbb{E}_{f^*}[q]) \leq \bar{U}(\bar{q})$, then we get that no q_0 in this interval brings a better expected payoff to the contractor than truthfully reporting $\bar{q}$.
We then want to show for any $q_0 \in [\frac{1-\delta}{1+w}\bar{q}, \frac{1+\delta}{1-w}\bar{q}]$ that $\mathbb{E}_{f^*}[q] \leq \bar{q}$, or equivalently that:

$$F((1+w)q_0) \cdot q_0 \leq \int_{(1-\delta)\bar{q}}^{(1+w)q_0} qdF(q). \tag{27}$$

First note that for $q_0 \leq (1-\delta)\bar{q}$, $\int_{(1-\delta)\bar{q}}^{(1+w)q_0} qdF(q) \geq \int_{(1-\delta)\bar{q}}^{(1+w)q_0} q_0 dF(q) = F((1+w)q_0) \cdot q_0$. Now, supposing $q_0 \geq (1-\delta)\bar{q}$ we can decompose the left-hand side in (27) as follows:

$$\int_{(1-\delta)\bar{q}}^{(1+w)q_0} q dF(q) = \int_{(1-\delta)\bar{q}}^{q_0} q dF(q) + \int_{q_0}^{2q_0-(1-\delta)\bar{q}} q dF(q) + \int_{2q_0-(1-\delta)\bar{q}}^{(1+w)q_0} q dF(q) \tag{28}$$

$$= \int_0^{q_0-(1-\delta)\bar{q}} [(q_0-\epsilon)\cdot f(q_0-\epsilon) + (q_0+\epsilon)\cdot f(q_0+\epsilon)]d\epsilon + \int_{2q_0-(1-\delta)\bar{q}}^{(1+w)q_0} q dF(q). \tag{29}$$

Where the two first parts of the integral are merged through a change of variable, resp. $\epsilon = q_0 - q$ and $\epsilon = q - q_0$. To characterize this first term in (29), consider $\epsilon \in [0, q_0 - (1-\delta)\bar{q}]$ and note that from the symmetry of f around $\bar{q}$ we have $f(q_0-\epsilon) = f(2\bar{q} - q_0 + \epsilon)$. Moreover, knowing $q_0 < \frac{1+\delta}{1+w}\bar{q} < \bar{q}$ we obtain that $q_0 - \epsilon < q_0 + \epsilon < 2\bar{q} - q_0 + \epsilon$ and therefore since F is single-peaked we know that $f(q_0-\epsilon) = f(2\bar{q}-q_0+\epsilon) \leq f(q_0+\epsilon)$. Thus we obtain:

$$(q_0-\epsilon)\cdot f(q_0-\epsilon) + (q_0+\epsilon)\cdot f(q_0+\epsilon) = q_0(f(q_0-\epsilon) + f(q_0+\epsilon)) + \epsilon((f(q_0+\epsilon) - f(q_0-\epsilon))$$
$$\geq q_0(f(q_0-\epsilon) + f(q_0+\epsilon)).$$

Then, plugging this inequality into (29) we obtain:

$$\int_{(1-\delta)\bar{q}}^{(1+w)q_0} q dF(q) \geq q_0 \underbrace{\int_0^{q_0-(1-\delta)\bar{q}} (f(q_0-\epsilon) + f(q_0+\epsilon))d\epsilon}_{=\int_{(1-\delta)\bar{q}}^{2q_0-(1-\delta)\bar{q}} f(q)dq} + \underbrace{\int_{2q_0-(1-\delta)\bar{q}}^{(1+w)q_0} q dF(q)}_{\geq q_0 \int_{2q_0-(1-\delta)\bar{q}}^{(1+w)q_0} f(q)dq}$$
$$\geq q_0 \int_{(1-\delta)\bar{q}}^{(1+w)q_0} f(q)dq = F((1+w)q_0)\cdot q_0.$$

We have then established the inequality (27), which implies (as detailed above) that no $q_0 \in [\frac{1-\delta}{1+w}\bar{q}, \frac{1+\delta}{1-w}\bar{q}]$ brings a better payoff to the contractor than reporting truthfully $\bar{q}$.

Case 4 This case corresponds to reported expected productions such that the lower bound of the insurance range is within the support of F: $(1-\delta)\bar{q} < (1-w)q_0 < (1+\delta)\bar{q}$, or equivalently $q_0 \in]\frac{1-\delta}{1-w}\bar{q}, \frac{1+\delta}{1-w}\bar{q}[$. We have already shown through the three previous cases that $q_0 = \frac{1-\delta}{1-w}\bar{q}$ brings a better payoff than any other $q_0 \notin]\frac{1-\delta}{1-w}\bar{q}, \frac{1+\delta}{1-w}\bar{q}[$, therefore the (globally) optimal report of expected production necessarily lies within the interval $[\frac{1-\delta}{1-w}\bar{q}, \frac{1+\delta}{1-w}\bar{q}[$.

The contractor's expected payoff on this interval is expressed as:

$$\Pi_i(b, f, q_0) = \int_{(1-\delta)\bar{q}}^{(1-w)q_0} \bar{U}(q)dF(q) + (1 - F((1-w)q_0))\cdot \bar{U}(q_0).$$

Within this interval, the corresponding derivative with respect to q_0 is expressed as:

$$\frac{\partial \Pi_i(b,f,q_0)}{\partial q_0} = (1-w)\left[\bar{U}((1-w)q_0) - \bar{U}(q_0)\right] f((1-w)q_0) + (1-F((1-w)q_0))\bar{U}'(q_0) \tag{30}$$

$$= U'(q_0) f((1-w)q_0)\left[\frac{1-F((1-w)q_0)}{f((1-w)q_0)} - (1-w)\cdot\frac{\bar{U}(q_0) - \bar{U}((1-w)q_0)}{BarU'(q_0)}\right]. \tag{31}$$

Note that since $\bar{U}'(q_0)f((1-w)q_0) > 0$, $\frac{\partial \Pi_i(p,f,q_0)}{\partial q_0}$ has the same sign as the term in brackets in (31), that we further denote $M(q_0)$. Then, any interior optimum within this interval, denoted by q_0^*, must satisfy the FOC:

$$M(q_0^*) \equiv \frac{1-F((1-w)q_0^*)}{f((1-w)q_0^*)} - (1-w)\cdot\frac{\bar{U}(q_0^*) - \bar{U}((1-w)q_0^*)}{\bar{U}'(q_0^*)} = 0. \tag{32}$$

Finally, any optimal reporting q_0^* (for a given pair (b,f)) satisfies either $q_0^* = \frac{1-\delta}{1-w}\bar{q}$ or the first order condition (32). The corresponding set of optimal reports, denoted next by $Q_i^*(b,f)$, can be further characterized when assuming:

- The distribution F is such that the function $q \mapsto \frac{1-F(q)}{f(q)}$ is continuously decreasing.[50]
- The PDF f is continuous on $\mathbb{R}_+$, or to put it otherwise the density f is vanishing at the bounds of its support: $\lim_{q\to(1-\delta)\bar{q}} f(q) = 0$.

Let us first consider the case of a risk-neutral contractor. In such a case, we use the notation $M^{RN}(q_0)$ for the function $M(q_0)$. If U is linear, then $\bar{U}$ is also linear and $\bar{U}(q_0) - \bar{U}((1-w)q_0) = wq_0\bar{U}'(q_0)$ and we have consequently:

$$M^{RN}(q_0) = \frac{1-F((1-w)q_0)}{f((1-w)q_0)} - (1-w)wq_0.$$

From the first assumption above, $M^{RN}(q_0)$ is decreasing in q_0 for any $w \in]0,1[$, and therefore $M^{RN}(q_0^*) = 0$ admits at most one solution. Moreover, since F is symmetric and single peaked we have that $f(\bar{q}) \geq \frac{1}{2\delta\bar{q}}$. Therefore:

$$M^{RN}\left(\frac{1}{1-w}\bar{q}\right) = \frac{1-F(\bar{q})}{f(\bar{q})} - w\bar{q} \leq \bar{q}(\delta - w) < 0.$$

[50] This assumption is stronger than most often needed, in order to cover any potential value taken by w: we actually only need the function $M(q_0)$ defined in Eq. (32) to be continuously decreasing in q_0 on the interval $]\frac{1-\delta}{1-w}\bar{q}, \frac{1+\delta}{1-w}\bar{q}[$.

Then there is a unique global optimal which necessarily belongs to the open interval $]\frac{1-\delta}{1-w}\bar{q}, \frac{1}{1-w}\bar{q}[$. This optimum denoted next q_0^{RN} is characterized as the solution of $M^{RN}(q_0^{RN}) = 0$ and thus does not depend on b.
In the general case, for any risk-averse contractor with the concave utility function U, we have $\bar{U}(q_0) - \bar{U}((1-w)q_0) \geq wq_0\bar{U}'(q_0)$ and therefore that $M(q_0) \leq M^{RN}(q_0)$ for any q_0 (with strict inequalities if U is strictly concave). If $q_0 > q_0^{RN}$, then $M(q_0) \leq M^{RN}(q) < 0$ which implies that $q_0^* \notin Q_i^*(b, f)$. Overall, for any concave utility function U, any optimum $q_0^* \in Q_i^*(b, f)$ is below the optimum with a risk-neutral contractor: $q_0^* \leq q_0^{RN}$ and the inequality is strict if U is strictly concave. In other words, any risk averse strategic contractor always overestimate its production less than a risk neutral strategic contractor.
In addition, note that the second assumption above (the continuity of f) implies that $\lim_{q_0 \to \frac{1-\delta}{1-w}\bar{q}} \frac{1-F((1-w)q_0)}{f((1-w)q_0)} = +\infty$ which further implies that:

$$\lim_{q_0 \to \frac{1-\delta}{1-w}\bar{q}} M(q_0) = +\infty$$

and therefore that the derivative of the contractor's payoff is positive (and infinite) at the lower bound $\frac{1-\delta}{1-w}\bar{q}$. The potential corner solution $q_0^* = \frac{1-\delta}{1-w}\bar{q}$ is then ruled out and any global optimum necessarily satisfies $M(q_0^*) = 0$.

Last, we assume the contractor's utility function is such that $\bar{U}$ is a CRRA utility function. The first order condition (32) simplifies to:

$$M_F(q_0^*; w, \gamma) \equiv \frac{1 - F((1-w)q_0^*)}{f((1-w)q_0^*)} - (1-w)q_0^* \cdot K(w, \gamma) = 0 \tag{33}$$

where $K(w, \gamma) = \frac{1-(1-w)^{1-\gamma}}{1-\gamma}$ if $\gamma \neq 1$ and $K(w, \gamma) = \log(1-w)$ if $\gamma = 1$. Note that $K(0, \gamma) = 0$ and $\frac{\partial K(w,\gamma)}{\partial w} = \frac{1}{(1-w)^\gamma} > 0$, therefore $K(w, \gamma) \geq 0$ for any pair (w, γ). Moreover $\frac{1-F(q)}{f(q)}$ is strictly decreasing on $](1-\delta)\bar{q}, \bar{q}[$, then $M_F(\cdot; w, \gamma)$ is strictly decreasing as well. Then Eq. (33) admits a single solution on this interval. Overall, we obtain that $Q_0^*(p)$ is a singleton and does not depend on p. Let q_0^* denote the global optimum.
We now are able to derive the following comparative statics on q_0^* from (33):

1. $K(w, \gamma)$ is increasing in γ and then $M_F(q_0; w, \gamma)$ is decreasing in γ for every q_0. Therefore, the optimal report q_0^* decreases with γ: the more risk averse firms are, the less they overestimate their production.

2. Consider two distributions F_1 and F_2 (on the same support), with F_1 less risky than F_2 in the sense that $\forall q \leq \bar{q}$, $\frac{f_1(q)}{1-F_1(q)} < \frac{f_2(q)}{1-F_2(q)}$. Then $M_{F_1}(q_0; w, \gamma) > M_{F_2}(q_0; w, \gamma)$ for any $q_0 \in]\frac{1-\delta}{1-w}\bar{q}, \frac{1}{1-w}\bar{q}[$ (the interval where the optima are to be found), and consequently the solution to $M_{F_1}(q_0; w, \gamma) = 0$ is larger than the solution to $M_{F_2}(q_0; w, \gamma) = 0$: if production is less risky, then firms overestimate more their expected production.

3. Assuming $\gamma \geq 1$, $K(w, \gamma)$ is non-increasing in w, and therefore $(1-w)q_0 \cdot K(w, \gamma)$ is strictly decreasing in w. In addition, since $\frac{1-F(q)}{f(q)}$ is decreasing on $](1-\delta)\bar{q}, \bar{q}[$, we also have $\frac{1-F((1-w)q_0)}{f((1-w)q_0)}$ decreasing in w for $q_0 \in]\frac{1-\delta}{1-w}\bar{q}, \frac{1}{1-w}\bar{q}[$. Then $M_F(q_0; w, \gamma)$ is strictly decreasing in w on the interval containing q_0^*, and therefore the greater is w the greater is the solution to (32): the larger the insurance range is, the more firms overestimate their production if $\gamma \geq 1$.

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